MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. H. JEANS, PRESIDENT, in the Chair.

William Ernest Allen, Lieut., R.N.R., 19 Manor Lane Terrace, London, S.E. 13 ;
William Harry Coombs, F.R.G.S., A.I.N.A., Royal Thames Yacht Club, Knightsbridge, London, S.W. ;
Alexander William Fenton, Thornwood and Broughton Nautical Schools, 4 and 5 Carlisle Avenue, London, E.C. 3 ;
Mohamed Reda Madwar, B.Sc., Ph.D., A.M.I.C.E., Helwân Observatory, Egypt ;
Thomas Harvey Walmsley, F.R.G.S., c/o The Naraguta Tin Mines Ltd., Jos, Nigeria ; and
Hugh Montague Wood, 5 Carlisle Avenue, Fenchurch Street, London, E.C. 3,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Redmond Barton Cafferata, Villa Natacha, Bvd. Carnot, Nice, A.M., France (proposed by E. D. O'Connor) ;
Henry William Mountcastle, Ph.D., Professor of Physics and Astronomy, Western Reserve University, Cleveland, Ohio, U.S.A. (proposed by J. J. Nassau) ; and
S. Lynn Rhorer, 26 Luckie Street, Atlanta, Georgia, U.S.A. (proposed by Harlow Shapley).

Fifty-three presents were announced as having been received since the last meeting, including, amongst others :—

International Research Council: First Report of the Commission appointed to further the study of Solar and Terrestrial Relationships.

Stroobant (Paul): Les Observatoires Astronomiques et les Astronomes.

Celestial Photographs.

The following have been added to the list of Celestial Photographs reproduced by the Society for sale to the Fellows :—

R.A.S. Ref. No.

369. Mars (and San Jose), 1924 Nov. 9, (*a*, *c*) Violet, (*b*, *d*) Infra-red. Wright, Lick Obs.
371. Solar Surface, 1885 June 2, Janssen.
372. Great Bolide, 1923 Sept. 12, Klepesta, Prague.
373. Exploding Andromedid Meteor, 1895 Nov. 23, Butler.
374. Solar Eclipse Track, 1715 April 22, Halley.
375. Solar Eclipse Track, 1724 May 11, Halley.

Gift for Endowment of an Annual Lectureship.

The Treasurer has received the following letter from the President :—

My dear Treasurer,—The Royal Astronomical Society has no lectureship or other endowment such as makes it possible in many Societies for an eminent person from abroad to be invited to give a lecture on his own work. It would give me great pleasure if Council should see fit to accept the following offer :—

I offer the sum of £1000 (a thousand pounds) for the endowment of a "Foreign Lectureship," the capital to be kept intact, and the income to be used for an annual lecture on some subject of interest to astronomers, preference being given to a lecturer normally resident outside the British Isles. I had thought that in particular the medallist of the year, when resident abroad, might frequently be invited to be Foreign Lecturer also,—it would give him an extra inducement to come in person to receive his medal. When no suitable Foreign Lecturer was available, Council could of course invite one of their own Fellows or someone in some cognate science, geophysics, or even physics of the right kind.

If Council accepts this offer and if, after fair trial of the Foreign Lectureship, they consider the money could be used better in other ways, I should of course wish this to be done. I should prefer (but without creating any legal obligation) that the income should be used

for some specific purpose, and not merged in the general funds of the Society, but quite realise this might not be possible.

Will you be so good as to bring this matter before Council?—Yours sincerely,

J. H. JEANS.

Lt.-Col. F. J. M. STRATTON,
Treasurer R.A.S.

The Council has gratefully accepted this munificent offer, and has appointed a small committee to consider the best way of carrying out the wishes of the donor.

The Solar Cluster: Further Note. By Professor R. A. Sampson, F.R.S.

I gather that some difficulty is felt in accepting the description of the Solar Cluster put forward in my paper of *M.N.*, 1926 April, on the ground of the incompleteness and possible selective character of the material. The following note may assist in clearing the position.

The stars dealt with, collected according to *apparent visual magnitude* and spectral class, reach the following numbers :—

TABLE I.

Vis. Mag.	B.	A.	F.	G.	K.	M.	Total.	*Cf.* Whole Sky.
m Up to 1·4	1	4	1	1	3	2	12	18
m m 1·5–2·4	4	8	2	0	3	1	18	56
2·5–3·4	4	20	7	8	21	4	64	172
3·5–4·4	8	34	29	32	50	7	160	550
4·5–5·4	2	46	52	48	48	7	203	1,800
5·5–6·4	4	23	75	69	59	17	247	4,800
6·5–7·4	1	9	30	93	25	2	160	14,000
7·5–8·4	0	4	15	92	60	6	177	30,000
8·5 and beyond	0	4	6	66	65	19	160	
Total	24	152	217	409	334	65	1201	

The approximate number of stars upon the whole sky, to the same limits of magnitude, is added. Nearly all the parallax stars are in North Declination, so that the comparison would be made more justly with half the sky; but this is immaterial. The feature that provokes remark is that the ratio of the whole number of stars to those of ascertained parallax grows progressively greater and greater until at the 8th magnitude it is more than 100 to 1. It is therefore open to suppose that were these missing parallaxes supplied, a system would appear utterly different to that which I have described, and showing perhaps no sign of clustering.

The only answer open at present is to show how the known stars of different apparent magnitudes are actually distributed in distance. This is seen in the following table.

TABLE II.

	Parsecs.	1. m To 1·4.	2. m 1·5–2·4.	3. m 2·5–3·4.	4. m 3·5–4·4.	5. m 4·5–5·4.	6. m 5·5–6·4.	7. m 6·5–7·4.	8. m 7·5–8·4.	9. m 8·5 on.
	1–10	..	..	..	..	..	..	..	..	..
	11–20	..	..	..	..	..	..	..	..	..
	21–30	..	1	..	..	..	..	..	..	..
	31–40	..	1	..	1	1	1	..	..	..
	41–50	..	..	..	1	1	..	..	..	..
B	51–60	..	..	..	..	..	..	..	..	..
	61–70	..	..	..	2	..	..	..	..	..
	71–80	..	..	..	1	..	..	..	..	..
	81–90	..	..	1	..	..	..	..	..	..
	91–100	..	..	..	1	..	..	..	..	..
	Over 100	1	2	3	2	..	3	1	..	..
	1–10	3	1	1	..	..	..	..	..	1
	11–20	1	2	5	2	3	..	1	..	..
	21–30	..	3	4	5	3	3	1	1	..
	31–40	..	1	1	11	4	3	3	..	..
	41–50	..	..	2	1	8	2	2	1	..
A	51–60	..	1	2	4	1	1	..	..	..
	61–70	..	..	..	2	2	1	..	..	..
	71–80	..	..	..	..	3	2	..	..	..
	81–90	..	..	3	3	6	1	..	..	1
	91–100	..	..	..	..	3	1	..	..	..
	Over 100	..	..	2	6	13	9	2	2	2
	1–10	1	..	..	..	..	..	..	..	..
	11–20	..	1	1	8	11	10	2	1	1
	21–30	..	..	1	6	17	22	2	1	..
	31–40	..	..	1	5	9	11	4	4	..
	41–50	..	..	1	2	4	12	4	1	..
F	51–60	..	..	..	1	3	2	2	1	2
	61–70	..	..	..	2	2	2	2	..	..
	71–80	..	..	..	1	2	3	1	1	..
	81–90	..	..	1	1	..	4	..	..	..
	91–100	..	..	..	..	..	..	..	1	..
	Over 100	..	1	2	3	4	9	13	4	2

TABLE II.—*continued.*

	Parsecs.	1. m To 1·4.	2. m 1·5–2·4.	3. m 2·5–3·4.	4. m 3·5–4·4.	5. m 4·5–5·4.	6. m 5·5–6·4.	7. m 6·5–7·4.	8. m 7·5–8·4.	9. m 8·5 on.
	1–10	..	..	2	7	7	2	2	..	..
	11–20	..	..	..	10	10	17	13	13	9
	21–30	..	..	1	1	9	15	22	16	11
	31–40	..	..	1	2	3	13	17	18	9
	41–50	..	..	..	2	8	5	11	9	6
G	51–60	..	..	..	2	1	5	7	7	6
	61–70	..	..	..	2	..	3	1	6	5
	71–80	1	..	..	1	3	1	6	3	4
	81–90	..	..	..	..	1	1	2	5	5
	91–100	..	..	..	2	1	..	4	1	1
	Over 100	..	..	4	3	5	7	8	14	10
	1–10	1	..	..	4	4	9	1	5	12
	11–20	1	..	2	3	2	6	12	21	15
	21–30	1	..	2	4	2	10	4	14	7
	31–40	..	1	1	10	5	2	3	7	13
	41–50	..	..	5	5	3	8	..	6	3
K	51–60	..	..	1	6	5	3	1	..	2
	61–70	..	..	3	2	3	2	..	2	2
	71–80	..	..	1	3	4	4	..	..	1
	81–90	..	..	2	3	5	2	1	..	3
	91–100	..	1	..	2	2	1	..	1	1
	Over 100	..	1	4	8	13	12	3	4	6
	1–10	..	..	..	..	..	..	..	..	10
	11–20	..	..	..	..	..	..	..	..	7
	21–30	..	..	1	..	..	1	..	..	..
	31–40	1	..	..	1	1	2	..	1	1
	41–50	..	1	..	..	..	1	..	..	..
M	51–60	..	..	1	1	..	2	..	..	..
	61–70	..	..	1	..	..	..	..	..	..
	71–80	..	..	..	..	..	1	..	..	..
	81–90	1	..	..	..	1	2	..	..	..
	91–100	..	..	..	1	1	1	..	..	..
	Over 100	..	..	1	4	4	7	2	..	1

In this table the stars of parallax less than ″·01 are enumerated in sum only; owing to the presence of accidental error, such parallaxes possess little meaning; as shown in the *Addendum* to my former paper they are, as likely as not, merely background stars which should not be present in the list. Though the numbers of them appearing in some of

the groups are considerable, the star density per unit volume that they imply is in all cases negligible.

To include a more complete collection of stars of the 5th, 6th . . . magnitudes would be in effect to combine the numbers of Table II. into a mean, with progressively greater and greater weight given to the higher magnitudes. I can see in these lists no sign that the result would differ in any vital respect from that already arrived at. The distribution in each magnitude appears to me substantially the same. If the stars here considered are a fair random selection from each magnitude, I would conclude that after their rearrangement according to luminosity or mass the description of the cluster given on p. 400 might stand unaltered, though the actual numbers indicative of the structures of the groups, as given in Schedule I., p. 404, might be varied materially.

This conclusion, however, depends on a proviso. It is necessary that the samples of each magnitude imperfectly represented should be truly characteristic of the whole. It may perhaps be surmised that parallax workers, taking an indication from proper motions, have chosen in these first lists for all magnitudes the stars that promised the biggest parallaxes, that is to say, besides any exclusion of parallaxes less than the probable error (*cf. M.N.*, 1926 April, p. 410) they have exercised a definite and continuous bias in favour of the smaller distances, and thereby produced the semblance of a law of clustering, which is illustrated, for example, in the columns for mag. 5 and 6 in class F. Then all the detail and regularities exposed in my former paper might be merely the expression and the product of a formula of calculation reproducing in different guises this semblance of a law.

In reply to this, I would point out that while the distribution in each class is very much the same for all magnitudes, it becomes quite different if we pass from one class to another—for example, from F to A, or from G to K. It does not therefore appear to be the result of the observer's selection. But this is in fact the same point that I dealt with in my other paper (p. 397, Table I.), only viewed from a different angle. It is there shown that when the stars are arranged in groups of equal mass, the points at which the thinning out of the groups becomes sensible as distance from the centre increases correspond to progressively different visual magnitudes for each class and each group within a class. The supposed bias certainly exists in some degree ; and even though it is a misleading guide, it would produce the same effect as a true clustering so far as it is operative. But the whole effect shows a strong discrimination between the different classes. If anyone attributes the whole effect to selection in the observing lists, it will be necessary to show how this discrimination between the classes has arisen from that selection.

It may not, however, be possible to produce conviction by such presumptions. The matter would become much clearer by an analysis of parallaxes spectroscopically determined ; these are more numerous and they are free from any imputation of bias. But if the clustering is real and not a trivial consequence of the selection of observing lists, other features of relation will certainly emerge, as in motions connected with the distances, and a search should be made for that purpose. Until this is done it is not unreasonable to suspend judgment.

Stellar Opacity and the Atomic Weight of Stellar Matter.

By J. H. Jeans.

1. The present paper brings forward certain considerations which suggest that the atomic weight of the matter in stellar interiors is probably far higher than has commonly been supposed. As a first and immediate consequence, the stellar opacity coefficient must be greater than its usual theoretically estimated value, and it seems likely that the observed and calculated values of this coefficient can in this way be brought into agreement.

General physical considerations are discussed in §§ 2–8; a mathematical investigation occupies §§ 9–15, and §§ 16–22 contain a discussion of the mathematical results obtained and of their bearing on the theory of stellar evolution.

2. There is now general agreement that stellar matter is in an advanced state of dissociation, the vast majority of the constituent units being free electrons which possess no degrees of freedom beyond those of translation. The ratio of the specific heats of stellar matter must consequently be near to $\gamma = 1{\cdot}666$; at any rate, until we come to densities so great that Boyle's law begins to fail.

The actual arrangement of a star, however, approximates to that of a mass of gas in adiabatic equilibrium with γ somewhat less than 1·333, and so certainly less than 1·666. It follows * that a star, if formed initially of a thoroughly mixed lot of atoms, would be stable for all radial displacements, so that there would be no convection currents, an exception possibly occurring in the regions quite close to the surface of the star, where the normal conditions fail. Thus in a star which was originally thoroughly mixed there would be a tendency for the heavier nuclei to sink toward the centre of the star.

This process of diffusion under gravity is, however, a very slow process, the "time of relaxation" being of the order of 10^{13} years. The process might be well advanced in a dwarf star, but could have hardly more than started in a giant star; at any rate, if the elements had been quite thoroughly mixed at the birth of the star. But it hardly seems likely that stellar matter can be so perfectly stirred, molecule by molecule, at the birth of a star, that a new-born star can be treated as thoroughly mixed in this way. More probably the star is originally a mixture of masses, some of high molecular or atomic weight and some of low. The former will rapidly find their way to the centre of the star by convection, so that we should expect to find a preponderance of heavy atoms at the star's centre even in its earliest stages.

3. In the present paper no attempt is made to estimate the extent to which light and heavy elements are separated in stellar interiors; all that we shall postulate is that a certain amount of separation is probable.

This postulate is in opposition to the conclusion of Rosseland † that "elements of different atomic weights would be expected to be nearly

* *Problems of Cosmogony*, § 191, p. 193.

† *M.N.*, **84** (1924), 722.

evenly distributed throughout the star." This conclusion was reached from a study of the electrical state of a star which seems to me to be based on very erroneous physical ideas. Clearly the free electrons in a star will tend, on account of their small mass, to diffuse towards the outer regions of the star, just as hydrogen and helium in the earth's atmosphere have diffused upwards as a consequence of their small molecular masses. Rosseland supposes that this process would stop "when the inward force on a free electron is equal to the inward force on a positive ion," and uses an equation of equilibrium which expresses the equality of these two forces. That this is not the true condition for equilibrium is shown by the analogous case of the earth's atmosphere, where the inward forces on molecules of, say, hydrogen and oxygen, are certainly not equal. The general impossibility of Rosseland's equations is seen at once on putting e (the charge on an electron)$=0$; they give the result that when all the electric charges disappear, the electric potentials become infinite.

The question which Rosseland's analysis actually examines is whether it is possible to have a configuration of equilibrium such that the forces on positive ions and negative electrons are everywhere equal. This is found to be possible only if the star has a positive charge E equal (in Rosseland's notation) to

$$\frac{G(M-m)}{e(Z+1)} \times \text{mass of star},$$

and calculation shows that for the sun this equals about 9×10^{11} coulombs. But the actual positive charge on a star is fixed by the condition that it shall be just adequate to restrain the negative electrons from escaping at the star's surface; thus the sun's positive charge is about

$$\frac{16Gm}{e} \times \text{mass of star},$$

or 4×10^{9} coulombs for the sun. Rosseland's solution refers to a star with a charge some 225 times as large as this actual charge, so that actual electric fields have only about ·0045 times the strength necessary to produce the effects described by Rosseland. For the present problem such effects may be disregarded altogether, and we may assume that atoms of highest atomic weight are likely to collect near the centre of a star.

4. One consequence of primary importance emerges at once. The only chemical elements known to us are those which exist at the surfaces of the stars, and on our earth which was originally drawn off from the surface of the sun. Thus all our acquaintances among the chemical elements have been selected by their tendency to rise to the surface on account of their lightness. It is natural to conjecture that there are elements at the centres of the sun and stars whose atomic weights are far higher than any known to us.

5. Considerations connected with the generation of energy confirm this conjecture. The sun generates about 2 ergs per gramme of its mass per second. Uranium (of atomic number 92) in equilibrium with its disintegration products yields about half this energy, or 1 erg per

gramme per second. With insignificant exceptions, all elements of atomic number less than 92 are known on earth, and show no energy-generating capacity at all comparable with that of the average matter of the sun. Thorium (90) generates 0·30 ergs per gramme per second, rubidium 0·0003, and potassium 0·0002 ergs per gramme per second; radium generates more energy per unit time while it lasts, but its life is too short for it to count astronomically. The possibility that terrestrial elements, if raised to solar temperatures, would generate energy on the same scale as the matter of the sun may be dismissed, for if matter had this property stars formed of it would be violently unstable.* Thus we seem driven to supposing that the main part, at least, of the sun's energy comes from elements of atomic number higher than 92.

The giant stars, radiating far more energetically per unit mass than the sun, provide a correspondingly stronger argument; the 600 ergs per gramme per second radiated by V Puppis may be expected to originate in elements of extremely high atomic weight.

6. The great majority of the spectral lines observed in the spectra of the sun and stars have been identified as originating in known terrestrial elements of atomic number less than 92, so that the heavier elements must probably be supposed to be confined to the inner parts of the sun and stars. This, of course, fits in with the argument of § 2. Further evidence is supplied by the observed radiating powers of the constituents of binaries; the less massive constituent, which may be supposed to have been formed from the outer layers of the parent star, is generally found to radiate less energy per unit mass than the more massive constituent which was presumably formed from the inner layers.

In this way we are led to picture stellar matter as consisting of a mixture of elements whose atomic numbers range from unity (hydrogen) to an upper limit about which nothing is known except that it is almost certainly higher than 92 (uranium). The heavier elements, which are also the elements of highest energy-generating capacity, have gravitated to the central regions of the star, leaving the elements of lower atomic weight in possession of the surface.

7. The hypothesis that a star's radiation is produced by the annihilation of its matter enables us to go even further. After some 7×10^{12} years, V Puppis, whose mass is now $20\odot$, will be reduced to a mass about equal to that of our sun. In this interval the matter which it now contains of atomic number less than 92 may either

(*a*) disappear by annihilation;
(*b*) disappear by transformation into matter of atomic number higher than 92; or
(*c*) remain in the star as matter of atomic number less than 92.

Terrestrial experience shows that matter of atomic number less than 92 annihilates itself very slowly, if at all. A rate of annihilation such that half the matter disappeared in 10^{14} years would involve the generation of 0·29 ergs per gramme per second, which is about the energy-

* *M.N.*, **85** (1925), 914.

generation of thorium. Thus we may suppose that in 7×10^{12} years there is no appreciable annihilation of matter of atomic number less than 92.

The loss by transformation into elements of atomic number higher than 92 must be slight, and more probably zero ; there is no suspicion at all that this process takes place on the earth, and elements of atomic number above 92 would probably make themselves noticed, immediately they were formed, by their high capacity for generating energy.

Thus we may reasonably conclude that matter of atomic number less than 92 persists without appreciable loss or change during the life of a star. A star of mass $20\odot$ can contain no more matter of atomic number below 92 than will survive after 7×10^{12} years in the then star of mass about equal to our sun. This latter mass accordingly fixes an upper limit to the total amount of matter of atomic number less than 92 which can exist in the present giant star. It is then clear that the great majority of atoms in a giant star must be of atomic number above 92.

8. We have now found reasons for supposing that both the atomic number and the generation of energy per unit mass will increase as we pass from a star's surface to its interior. We have further found reasons for supposing that the majority of atoms in a giant star are of atomic number above 92.

We proceed to examine the emission of radiation from a star with special reference to these suppositions. The suppositions are in no way essential to the theoretical discussion; they are regarded merely as providing hypotheses to be tested, and possible effects to be considered.

9. General physical theory shows that the coefficient of stellar opacity must consist in the main of two terms, one arising from "free-free" encounters and the other from absorption by electrons initially bound. Other contributions are almost certainly small in comparison with these two.* If we denote the "free-free" term by k_{ff}, and put the second term equal to Fk_{ff}, then the total opacity-coefficient k is given, with sufficient accuracy for our present purpose, by

$$k=(1+F)k_{ff} \quad . \quad . \quad . \quad . \quad (1)$$

The theory of Kramers † gives

$$k_{ff}=\frac{\kappa\rho}{\mu T^{3\cdot5}} \quad . \quad . \quad . \quad . \quad (2)$$

where ρ, T denote density and temperature as usual, μ is the effective molecular weight of the mixture of free electrons and ionised atoms which constitute stellar material, and κ is given by

$$\kappa=\frac{16\pi^2}{3\sqrt{3}}\frac{e^6u_0}{m_H{}^2aC^4mh}\left(\frac{N^2}{A}\right) \quad . \quad . \quad . \quad (3)$$

Here N, A are the atomic number and atomic weight of the atoms, u_0 is the average velocity of a free electron at 1° abs, m_H and m denote the masses of the hydrogen atom and electron respectively, and all the

* Eddington, *M.N.*, **84** (1924), 120.
† *Phil. Mag.*, **46** (1923), 836.

other symbols have their usual meanings. On inserting well-established numerical values, equation (3) takes the form

$$\kappa = 2{\cdot}23 \times 10^{24}\left(\frac{N^2}{A}\right) \quad . \quad . \quad . \quad . \quad (4)$$

It has been usual to assume a conjectural value for N^2/A and to treat this as constant throughout the star. But, as we have just found reasons for supposing that N^2/A may vary substantially through a star, we retain N^2/A in our equations and refrain from treating it as a constant.

The complete evaluation of k requires the evaluation of F. Eddington * has proposed for this a value of the form

$$F = \frac{h\nu}{RT} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where ν is a calculable frequency and h, R, T have their usual meanings; he has given calculations which suggest that F cannot differ greatly from 9.†

10. This completes the theoretical evaluation of k, and it now becomes possible to estimate how much radiation ought to be emitted by a given star. Eddington has estimated ‡ that the observed radiation from Capella demands a value of k some eight times as great as is given by the theoretical formula if N and A are given the values appropriate to iron.

This result is based on the supposition that the generation of energy is distributed in a special way ($k\eta$=cons.) which involves a substantial concentration of the energy-generation in the central regions of the star. In the present paper it is found that if the generation of energy were uniformly distributed throughout the star the discrepancy would be represented by a factor of 21·4 as against Eddington's estimate of 8.

Eddington has further estimated § that in the extreme case in which the whole generation of energy is concentrated at the centre of the star, the discrepancy is reduced by a factor of 2·5 (or 0·97 m.) below that corresponding to his special assumption ($k\eta$=cons.) and so to a factor of about 3·2.

11. Other factors remain to be taken into account. Rosseland ‖ has shown that the distribution of the radiation over different wavelengths increases the discrepancy by a factor which Eddington estimates as 1·5, while Milne finds that Eddington's value for (1+F) is too small by a factor of about 3.¶ These two corrections can both be allowed for by assigning to 1+F a value of 20 in the place of Eddington's value of 10. The effect of this is to reduce all the discrepancies so far mentioned by one-half. Thus for a Capella in which the whole generation of energy is concentrated at the centre, the outstanding discrepancy is represented by a factor of only 1·6. This is calculated on the supposition that $N^2/A\mu$ has the value appropriate to iron (5·4); the whole discrepancy

* *M.N.*, **84** (1924), 108.
† *Loc. cit.*, pp. 110, 111.
‡ *Loc. cit.*, 121.
§ *M.N.*, **85** (1925), 408.
‖ *M.N.*, **84** (1924), 525.
¶ *M.N.*, **84** (1925), 750. It is only in rather favourable cases that the factor is as high as 3 with ordinary atomic weights, but it seems likely to be substantially higher than 3 for atoms of high atomic weights.

can accordingly be removed by supposing that $N^2/A\mu$ has a value equal to 1·6 times that of iron (8·6), so that the average atomic weight is about equal to that of silver or palladium. This is well below the atomic weight which we have been led to regard as probable, so that we conclude that theory and observation can be reconciled, in the case of Capella at least, without having recourse to the extreme assumption that the whole energy-generating capacity is concentrated at the centre of the star.

12. The whole problem is rather more intricate than would appear from this very cursory discussion. In the present paper we examine it afresh, avoiding any supposition as to the constancy of $k\eta$ and extending it to stars other than Capella. The necessary mathematical analysis has already been given in my papers "On the Masses, Luminosity, and Surface Temperatures of the Stars"*; it is only necessary to repeat this analysis, evaluating all numerical constants as we proceed. Certain further refinements are, however, introduced in the process. It seems unnecessary to repeat the analysis in detail: this can be found in the earlier papers, and to facilitate comparison I have numbered the formulæ of the present paper non-consecutively, so that they agree with the numbers of the corresponding formulæ of the earlier papers.

As before, the two equations of equilibrium are

$$\frac{3CT^{\frac{1}{2}}}{a\kappa(1+F)}\left(\frac{R}{m_H}\right)\frac{q}{p}\frac{dq}{dr}=-\frac{\eta\rho}{r^2}\int_0^r 4\pi\rho r^2 dr \quad . \quad . \quad (12)$$

$$\frac{d}{dr}(p+q)=\frac{3C\gamma}{a\kappa}\left(\frac{R}{m_H}\right)\frac{T^{\frac{1}{2}}}{\eta}\frac{q}{p}\frac{dq}{dr} \quad . \quad . \quad . \quad (13)$$

To represent a possible general tendency for both N^2/A and η to increase as we pass inwards in a star, let us assume that

$$\kappa\eta=\kappa_0\eta_0 T^l \quad . \quad . \quad . \quad . \quad (14)$$

when l is a small positive constant. To avoid difficulties of the kind discussed in § 19 below, we shall suppose that equation (14) is not true throughout the star but only within a sphere of radius r', this sphere being large enough, nevertheless, to enclose the main part of the mass and also of the energy-generation of the star.

Inside the sphere of radius r' we may write equation (13) in the form

$$\frac{d}{dr}(p+q)=K\frac{q^{1+s}}{p}\frac{dq}{dr} \quad . \quad . \quad . \quad . \quad (15)$$

where $4s=\frac{1}{2}-l$ and

$$K=\frac{C\gamma}{(\frac{1}{3}a)^{s+1}(1+F)\kappa_0\eta_0}\left(\frac{R}{m_H}\right) \quad . \quad . \quad . \quad (16)$$

On writing λ for p/q, the ratio of gas-pressure to pressure of radiation, equation (15) takes the form

$$q\frac{\partial\lambda}{dq}=\frac{Kq^s}{\lambda}-(\lambda+1) \quad . \quad . \quad . \quad . \quad (62)$$

* *M.N.*, **85** (1925), 196, 304.

of which the solution is

$$\lambda=x\psi=x\left(1-\frac{sx^2}{4x^2+1}\right)\Psi'_0 \quad . \quad . \quad . \quad (71)$$

where $x^2=Kq^s$ and Ψ'_0 is the function of x tabulated already.*

The effective molecular weight μ, which has not so far appeared in our equations, will vary inside the star on account of variations both in the degree of ionisation and in the value of N^2/A. Let us suppose that

$$\mu=\mu_0 T^j$$

where j is a small constant. Then

$$\rho=\frac{p}{T}\mu\left(\frac{m_H}{R}\right)=A\lambda q^{\frac{1}{4}(3+j)} \quad . \quad . \quad . \quad (73)$$

where

$$A=(\tfrac{1}{3}a)^{\frac{1}{4}(1-j)}\mu_0\left(\frac{m_H}{R}\right) \quad . \quad . \quad . \quad . \quad (74)$$

With this value for ρ equation (12) becomes

$$\int_0^r 4\pi\rho r^2 dr=-\frac{r^2}{\gamma A\psi^2}q^{-\frac{1}{4}(3+j)}\frac{dq}{dr} \quad . \quad . \quad . \quad (74a)$$

whence, on differentiation with respect to r and replacing q^s by x^2/K, we obtain

$$\frac{1}{(1-j)\pi r^2}\frac{d}{dr}\left[\frac{r^2}{\gamma A\psi^2}\frac{d}{dr}\left(\frac{x^2}{K}\right)^{\frac{1-j}{4s}}\right]+A\psi\left(\frac{x^2}{K}\right)^{\frac{3+j}{4s}}=0 \quad . \quad (76)$$

As before, if we write

$$R^2=(1-j)\pi\gamma A^2\psi^3 K^{-(1+j)/2s}r^2, \qquad y=x^{\frac{1-j}{2s}}$$

the equation takes the form

$$\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{dy}{dR}\right)+y^{(3+2s+j)/(1-j)} \quad . \quad . \quad . \quad (77)$$

which is Emden's standard form with

$$n=\frac{3+2s+j}{1-j}.$$

The general solution is

$$y=fy_1, \qquad R=f^{-(1+s+j)/(1-j)}R_1 \quad . \quad . \quad . \quad (78)$$

where y_1, R_1 refer to Emden's standard solution.

13. From equation (74*a*) we now obtain

$$\frac{M_r}{\odot}=-\frac{4f^{-(s+2j)/(1-j)}K^{j/2s}}{\gamma\odot A^2(\pi\gamma)^{\frac{1}{2}}\psi^{\frac{1}{2}}(1-j)^{\frac{1}{2}}}R_1{}^2\frac{dy_1}{dR_1} \quad . \quad . \quad (78a)$$

This is true throughout the star up to the sphere $r=r'$. At this the values of M_r and $R_1{}^2dy_1/dR_1$ are proportional to (and practically equal

* *M.N.*, **85** (1925), 395.

to) the values at the surface of the star, so that equation (78*a*) is true if M_r is taken to be the total mass M of the star, while $R_1^2 dy_1/dR_1$ is evaluated at the star's surface. When $n=3$, Emden's tables give $-R_1^2 dy_1/dR_1=2{\cdot}015$ at the surface of the star. By interpolation we find that for $n=(3+2s+j)/(1-j)$ where j is small,

$$\log(-R_1^2 dy_1/dR_1)=0{\cdot}304-0{\cdot}065(2s+4j).$$

Inserting this numerical value, and also $\log(\frac{1}{4}\gamma\odot(\pi\gamma)^{\frac{1}{2}})=22{\cdot}183$, we obtain from equation (13)

$$\log\frac{M}{\odot}=\frac{j}{2s}\log K-2\log A-\tfrac{7}{2}\log\psi-\tfrac{3}{2}\log(1-j)$$
$$-\frac{s+2j}{1-j}\log f-21{\cdot}879-0{\cdot}065(2s+4j) \quad . \quad (79)$$

By a precisely similar argument, if E is the emission of the star in ergs, and T_e its effective temperature,

$$\sigma T_e^4=\frac{E}{4\pi r^2}=\frac{E}{4\pi R_1^2}(1-j)\pi\gamma A^2\psi^3 K^{-(1+j)/2s} f^{(2+2s+2j)/(1-j)} \quad . \quad (79a)$$

where again R_1^2 must refer to the surface of the star. From Emden's tables we obtain, by the same method as before,

$$\log R_1^2=1{\cdot}678+0{\cdot}50(2s+4j),$$

and on inserting this value, and also $\log(\gamma/4\sigma)=-3{\cdot}536$, we obtain

$$4\log T_e=\log E+\log(1-j)+2\log A+3\log\psi-\frac{1+j}{2s}\log K$$
$$+\frac{2+2s+2j}{1-j}\log f-5{\cdot}214-0{\cdot}50(2s+4j) \quad . \quad (80)$$

The elimination of f between equations (79) and (80) gives

$$(2+2s+2j)\log\left(\frac{M}{\odot}\right)+4(s+2j)\log T_e-(s+2j)\log E+\tfrac{1}{2}(1-j)\log K$$
$$+2(2+s)\log A+(7+4s+j)\log\psi+43{\cdot}758+49{\cdot}232s+53{\cdot}410j$$
$$+1{\cdot}260s^2+3{\cdot}916sj+4{\cdot}088j^2=0 \quad . \quad (80a)$$

in which $\log(1-j)$ has been replaced by its value $-0{\cdot}432j$.

14. We have supposed the sphere r' to be so chosen that the total mass and total generation of energy inside it are approximately equal to those of the whole star. Thus we may write

$$E=4\pi\eta' M_{r'}=\frac{4\pi\kappa_0\eta_0}{\kappa'}T'^4 M$$

where κ', T', η' are the values of κ, T, η at the surface $r=r'$. Eliminating $\kappa_0\eta_0$ between this and equation (16) we obtain

$$KE=\frac{4\pi G\gamma\odot}{(\frac{1}{3}a)^{s+1}(1+F)\kappa'}\left(\frac{R}{m_H}\right)T'^4\left(\frac{M}{\odot}\right).$$

Taking logarithms and inserting the numerical values

$$\log(\tfrac{1}{3}a)=-14\cdot594, \qquad \log\left[\frac{4\pi C\gamma\odot}{\tfrac{1}{3}a}\left(\frac{R}{m_H}\right)\right]=60\cdot214,$$

we obtain

$$\log K=-\log E+\log\left(\frac{M}{\odot}\right)-\log\left[\frac{\kappa'(1+F)}{T'^{l}}\right]+60\cdot214-14\cdot594s.$$

From equation (74) we obtain

$$\log A=\log\mu_0-11\cdot568+3\cdot648j.$$

To an adequate approximation we may take

$$\psi=(1-\tfrac{1}{4}s)\Psi_0$$

(*cf.* equation (71)), and therefore

$$\log\psi=\log\Psi_0-0\cdot108s.$$

We further have (*cf.* equation (4)),

$$\log\kappa=24\cdot349+\log(N^2/A)$$
$$\log E=35\cdot531-0\cdot4m$$

whence we obtain equation (80*a*) in the form

$$\begin{aligned}(1-j)\log\left[\frac{1+F}{T'^{l}}\left(\frac{N^2}{A}\right)'\right]&-(5+4s+3j)\log\left(\frac{M}{\odot}\right)+8(s+j)\log T_e\\&+0\cdot4m(1+2s+3j)+(8+4s)\log(\mu_0 T_e^{\,j})+(14+8s+2j)\log\Psi_0\\&-4\cdot692-5\cdot788s-6\cdot454j+1\cdot656s^2+7\cdot616sj+8\cdot176j^2 \quad . \quad . \quad (87)\end{aligned}$$

15. To represent the simple case of a star in which N^2/A, η and μ have uniform values throughout, we put $j=l=0$ and $s=\frac{1}{8}$, and obtain

$$\log\left[(1+F)\frac{N^2}{A}\right]=5\cdot5\log\left(\frac{M}{\odot}\right)+\log T_e+\tfrac{1}{2}m+8\cdot5\log\mu+15\log\Psi_0-5\cdot390 \quad (88)$$

The values for Capella are $M=4\cdot18\odot$, $T_e=5200$, $m=-0\cdot36$, $\Psi_0=0\cdot849$, and on further using Eddington's value of $\mu=2\cdot2$, we find

$$\log\left[(1+F)\frac{N^2}{A}\right]=3\cdot41.$$

The values assumed by Eddington ($N=26$, $A=56$, $1+F=10$) would make the left-hand number equal to 2·08, so that in this case the discrepancy between theory and observation would be represented by a logarithm difference of 1·33 and so by a factor of 21·4, the result already quoted. Other stars show somewhat different factors, that for the sun being 31·0 and that for Kruger 60A only 4·7.

16. Let us again take $\mu=2\cdot2$ and put $1+F=20$ for the reasons explained in § 11. Replace s by its value $\frac{1}{8}-\frac{1}{4}l$, and neglect squares of the small quantities l and j. Then equation (87) becomes

$$\log\left(\frac{N^2}{A}\right)'=C+Dl+Ej \quad . \quad . \quad . \quad . \quad (89)$$

where

$$C=5\cdot5\log(M/\odot)+\log T_e+\tfrac{1}{2}m+15\log\Psi_0-3\cdot781 \quad . \quad . \quad . \quad . \quad (90)$$

$$D=\log(T'/T_e)-\log T_e-0\cdot2m-2\log\Psi_0-\log(\mu_0 T_e^{\,j})-\log(M/\odot)+1\cdot344 \quad (91)$$

$$E=C+8\log T_e+1\cdot2m+2\log\Psi_0-3\log(M/\odot)-5\cdot153 \quad . \quad . \quad . \quad (92)$$

The values of C, D, E calculated for various stars are shown in the following table, together with some other quantities which prove to be important. The order of the stars is approximately that of the reliability of the data ($M/\odot$, $m-m_{Sun}$ and T_e) used in the computations. The quantity $\mu_0 T_e^{\,j}$, which is the effective molecular weight at the photosphere, is for the moment given the standard value 2·2, subject to correction later.

Star.	C.	$D-\log(T'/Te)$.	$\log(T_0/Te)$.	D′.	E.	$E-8\cdot5\log(T_0/Te)$.	$\frac{N^2}{A}(j=-0\cdot1)$.
Sun	2·27	−3·72	3·71	−0·01	33·13	1·60	19
α Cent. A	2·29	−3·63	3·64	+0·01	31·98	1·04	23
α Cent. B	2·17	−3·58	3·58	0·00	31·89	1·46	16
Capella A	2·11	−3·12	3·16	+0·04	24·30	−2·56	35
Capella B	2·19	−3·31	3·33	+0·02	26·48	−1·82	35
Sirius	2·25	−3·52	3·46	−0·06	29·17	−0·27	28
60 Krug. A	1·45	−3·91	4·00	+0·09	31·19	−2·81	8
60 Krug. B	1·67	−4·12	4·00	−0·12	33·52	−0·48	8
β Aur.	1·79	−3·34	3·41	+0·07	27·75	−1·23	12
Procyon	1·59	−3·44	3·53	+0·09	30·39	0·39	5
V Puppis	2·29	−3·24	3·20	−0·04	21·61	−5·61	107
Z Vulpec	1·75	−3·23	3·27	+0·04	24·86	−3·03	17

17. The values of N^2/A necessary to reconcile theory and observation on the supposition that η, μ, and N^2/A are uniform throughout the star are of course given by $\log(N^2/A)=C$. This gives the following values for N^2/A :—

Sun 186, α Cent. A 195, α Cent. B 148, Capella A 129, 60 Krug. A 28.

These are, however, all subject to correction from the circumstance that the value 2·2 assumed for μ is too small for these values of N and A.

18. Suppose next that η and μ are constant throughout the star, but not N^2/A. Then $j=0$, η is constant, and N^2/A varies as T^l.

On putting $j=0$ in equation (89) we obtain the value of N^2/A at points at which the temperature is T′. The value at the centre at which the temperature is T_0 is accordingly given by

$$\log\left(\frac{N^2}{A}\right)_{centre}=C+D'l \quad . \quad . \quad . \quad . \quad (93)$$

where

$$D'=D-\log(T'/T_e)+\log(T_0/T_e) \quad . \quad . \quad . \quad (94)$$

The quantity D′ is tabulated in the fifth column of the table; it is necessarily independent of T′. It is at once seen to be negligibly small

for all stars. Thus a concentration of heavy atoms at the centre of the star, as represented by a positive value of l, leaves the requisite value of N^2/A approximately unaltered at the star's centre, while, of course, reducing the value substantially in all other parts of the star.

In other words, we can reconcile theory with observation by supposing that $\log(N^2/A)$ has a maximum value C at the star's centre and falls off to any value we please at the surface. But, again, a correction has to be made since the assumed value $\mu=2{\cdot}2$ is too small for the values of N and A now under consideration.

19. Suppose next that the generation of energy is unevenly distributed through the star while μ and N^2/A are constant throughout. We now suppose that η varies as T^l, but we are not free to pass to the limit and take $T'=T_e$ and $r'=r$. If we did this, any, except quite infinitesimal values of l, would imply a negative generation of energy in the outer layer of the star, and our model would become too artificial to be of any scientific interest. We avoid this difficulty by taking the surface $r=r'$ well inside the star. With the value $l=\frac{1}{2}$ we may take $r'=\frac{1}{2}r$ without introducing any appreciable negative generation of energy,* and this value of r' satisfies the requisite condition that practically the whole mass and the whole generation of energy reside inside the sphere $r=r'$.

With this value for r', T_0/T' is equal to about 4.† If we neglect the small value of D', equation (89) now gives

$$\log(N^2/A)=C-0{\cdot}301 \quad . \quad . \quad . \quad . \quad (95)$$

so that the requisite values of N^2/A are about half those previously calculated. For instance, the factor of discrepancy for Capella, calculated in § 15, is reduced from 21·4 to 10·7, which practically agrees with Eddington's estimate of 8.

It is impossible to estimate what would be a reasonable value to assign to l. Instead of attempting to do this let us pass to the extreme limiting case examined by Eddington, in which the whole generation of energy is concentrated at the centre. The factor of reduction in this case is one of 10·7/3·2 beyond that already introduced into equation (95), so that equation (95) must be replaced by

$$\log(N^2/A)=C-0{\cdot}825 \quad . \quad . \quad . \quad . \quad (96)$$

and the values of N^2/A are about 15 per cent. of those calculated in § 17. Calculated values for N^2/A are:—

Sun 28; α Cent. 29, 22; Capella 19, 23; Sirius 27.
60 Krug. 4, 7; β Aur. 9; Procyon 6; V Puppis 29; Z Vulpec 8.

Since A is somewhat greater than 2N, the values of N, the atomic number, are rather more than twice the above numbers, and the values of A, the atomic weight, rather more than four times.

20. Finally, let us examine the effect of variations in μ as represented by the term Ej in equation (89). On account of increasing ionisation, μ must decrease on passing inwards in a star, but the decrease will

* *Cf.* Eddington, *M.N.*, **85** (1925), 409.
† Emden, *Gas Kugeln*, p. 80 (Table 7).

be balanced in part by the increasing atomic weight of the atoms. On the whole, however, μ will be less near the centre than near the surface of the star, so that j must be supposed negative, and as E is large this can cause a substantial reduction in N^2/A.

This may, however, be more than counterbalanced by an adjustment which must be made in the absolute value of μ. Let μ_c, μ_e be the values of μ at the centre and surface of the star, so that

$$\log\mu_c - \log\mu_e = j \log(T_0/T_e).$$

Our calculations have been made on the supposition that $\mu_e = 2{\cdot}2$; this would give

$$\log\mu_c = 0{\cdot}342 + j \log(T_0/T_e).$$

If μ_c has a value greater than this in the actual star, we must adjust our calculations by adding to $\log N^2/A$ a term (*cf.* equation (88))

$$8{\cdot}5[\log\mu_c - {\cdot}342 - j \log(T_0/T_e)].$$

Equation (96), modified by the addition of this term and the term in j, becomes

$$\log(N^2/A) = C - 0{\cdot}825 + 8{\cdot}5[\log\mu_c - {\cdot}342] + j[E - 8{\cdot}5 \log(T_0/T_e)] \quad (97)$$

21. The values of N^2/A obtained by taking $\mu_c = 2{\cdot}2$ (so that the term $[\log\mu_c - {\cdot}342]$ again drops out) and $j = -0{\cdot}1$ are given in the last column of the table on p. 570.

These values have still to be adjusted by the addition of the term to $\log(N^2/A)$

$$+8{\cdot}5 \log[\mu_c - {\cdot}342] \quad . \quad . \quad . \quad . \quad (98)$$

For many of the stars we may suitably assume that $\mu = 2{\cdot}2$, this value not being inconsistent with the values obtained for N^2/A. In such cases no adjustment need be made. But the values of N^2/A shown for the giant stars V Puppis, Capella A and B, and probably Sirius also, are too large for $\mu_c = 2{\cdot}2$ to be a possible value.* For these stars the value of $\log N^2/A$ must be increased by the term (98), so that the values of N^2/A will be substantially higher than those shown in the table.

Three points may be specially noticed.

I. Even when the adjustment has been made, the values of N^2/A are considerably smaller than the theoretical considerations advanced in §§ 2–8 would have led us to expect. This means that these considerations suffice to reconcile theory and observation, as regards the value of k, without passing to the extreme limiting case in which the whole generation of energy is concentrated at the centre of the star.

II. The adjustment for the value of μ_c leaves the order of the stars unaltered when arranged by descending values of N^2/A. Now this order follows very closely the order of age—the young giant stars,

* There are two causes of increase in μ_c, first that atoms of high atomic weight will be imperfectly broken up, and second, that even if they were fully broken up, the value of μ would be greater than 2·2. A certain amount must be offset through our assumed value for 1+F being probably too low for atoms of high atomic weight.

V Puppis, Capella, etc., are found to have high atomic weights, while the old dwarf stars, such as 60 Kruger and Procyon, have low atomic weights. This fits in with the hypothesis (§ 5) that stellar evolution is accompanied by an annihilation of the elements of highest atomic weight and a consequent gradual reduction of the mean atomic weight as the star ages.

The values of N^2/A shown in the last column of the table are calculated for the value $j=-0{\cdot}1$; this represents a ratio of about 2·4 in the highest and lowest values of μ occurring in the same star. This may be thought rather too large, but a smaller value of j would place the stars in substantially the same order when arranged by values ot N^2/A ; indeed, traces of this order can be quite definitely recognised in the values of N^2/A in the limiting case of $j=0$, which are given at the end of § 19.

III. Certain stars in the table show values of N^2/A which appear to be unduly small for the star's age. Conspicuous instances are the giant star Z Vulpec, β Aurigæ, and Procyon. Observationally these stars are noticeable as being unduly bright for their mass ; thus Z Vulpec is about eight times as luminous as V Ophiuc. of equal mass, β Aurigæ is about three times as luminous as Sirius, and Procyon about four times as luminous as α Centauri A. These stars are all on the extreme edge of the main sequence in the direction of " early " spectral type. According to the theory of stellar evolution which I recently put forward * the generation of energy in such stars ought to be less concentrated near the centre than in other stars, so that the value of N^2/A must be less reduced on account of central condensation of energy-generating capacity than for the rest of the stars in the table. When a suitable increase is made in the values of N^2/A, shown in the last column of the table, for these stars, they at once fall into their proper places in the sequence arranged according to age.

22. We have seen that a consistent theory of stellar radiation can be based on the suppositions that the atomic weights of the elements in stellar interiors are far higher than the atomic weights of terrestrial elements, and that they vary from star to star, the mean atomic weight decreasing as the star gets older. The acceptance of these hypotheses compels certain modifications in some of my earlier papers. In two papers † on " The Masses, Luminosities, and Surface-Temperatures of the Stars," I examined the relation between these quantities on the supposition that the mean value of N^2/A did not vary from star to star. The mathematical theory of those papers remains valid, but the numerical work becomes useless, being superseded by the numerical results of the present paper. My recently published " Theory of Stellar Evolution " ‡ also needs emendation in one particular. I supposed, as a conjectural hypothesis, that no generation of energy from matter could occur in completely ionised atoms. This hypothesis seemed somewhat artificial, but had the justification that it gave a definite " main-sequence " bounding the temperature-luminosity diagram on the side of high temperature, which fell at about the right place when the mean atomic number was supposed to be about 20.

* *M.N.*, **85** (Suppl. No. 1925), 914.
† *M.N.*, **85** (1925), 196, 394.
‡ *M.N.*, **85** (1925), 914.

When we suppose stellar atomic numbers to be of the order of 100 or 200, this hypothesis gives an entirely wrong position for the main sequence, and must, I think, be abandoned. Some other hypothesis must take its place and fix the position of the main sequence. A simple hypothesis, which seems to have much in its favour, is the hypothesis that all bound electrons in the same atom have the same chance of annihilation, while free electrons have no chance at all. This appears to fix the main sequence somewhere near the right place. The hypothesis is probably too simple to represent the ultimate truth, as an electron's chance of annihilation is likely to be a complicated function of the quantum numbers of its orbit; but it seems that some hypothesis on these lines must replace the still simpler earlier hypothesis which is now seen to be inadequate.

The Exact Equations of Radiative Equilibrium.

By J. H. Jeans.

1. Schwarzschild's original equation of radiative transfer of energy was exact, subject to the physical assumptions on which it was based. Approximate equations of radiative equilibrium have been deduced from this equation by Eddington, Milne, and others, and various theorems have been proved by the use of these approximate equations. The exact equations of radiative equilibrium which are given in the present note make it possible to examine whether the various theorems deduced from the approximate equations are valid or the reverse.

2. Schwarzschild's well-known equation of radiative transfer is

$$\frac{\partial \mathrm{I}}{\partial s}=\frac{\mathrm{E}\rho}{4\pi}-k\rho\mathrm{I} \qquad . \quad . \quad . \quad . \quad (1)$$

where I is the intensity of a beam of radiation, ds is an element of its path, and E is the emission of radiation per unit mass. For the present let us disregard the curvature of the star and imagine it arranged in parallel layers perpendicular to the axis of z. Putting $ds=-dz\sec\theta$ and $k\rho dz=-d\nu$, the equation becomes

$$\cos\theta\frac{\partial \mathrm{I}}{\partial \nu}=\mathrm{I}-\frac{\mathrm{E}}{4\pi k}.$$

The complete integral of this equation is

$$\mathrm{I}=\mathrm{A}e^{\nu\sec\theta}+\left(1-\cos\theta\frac{\partial}{\partial\nu}\right)^{-1}\left(\frac{\mathrm{E}}{4\pi k}\right) . \qquad . \quad . \quad (2)$$

3. The first term represents a part of I which increases or decreases exponentially with $\nu\sec\theta$. When $\sec\theta$ is positive A must be zero; when $\sec\theta$ is negative the term will be insignificant in the far interior of the star. This term is important only near the surface of the star, where it makes it possible to satisfy the boundary-condition that there shall

be no inward flow of radiation. The presence of this term constitutes what may be described as the " boundary disturbance."

Since $\sec\theta$ is greater than unity, the surface-disturbance falls off more rapidly than $e^{-\nu}$ as we pass into the star. Thus the greater part of the surface-disturbance must be outside the photosphere, which is approximately given by $\nu=1$.

Clearly, then, the store of energy involved in the surface-disturbance is quite insignificant. It may be regarded as a shell of radiation extending approximately from the surface of a star to its photosphere, and of temperature comparable to that of the photosphere. The " pulsation-theory " of Cepheid variation relies on this store of radiation to account for the observed difference in phase between the light-curve and the radial-displacement curve. The observed lag is about a quarter of a period, so that the requisite store of energy is comparable with that emitted by the star in a quarter-period. But the energy of the surface-disturbance, being only a few kilometres in thickness, is emitted in a tiny fraction of a second, and so is seen to be quite inadequate for this purpose. Dr. Reesinck * has challenged my recent analysis of the radiation from a pulsative star on grounds which amount to my having disregarded the energy of the surface-disturbance, but the foregoing consideration appears to me to provide a valid defence of my treatment, and the pulsation theory seems to me to fall accordingly.

4.† In regions so far inside a star that the surface-disturbance may be neglected, the value of I is given by the second term of equation (2), and so by

$$\mathrm{I}=\frac{\mathrm{E}}{4\pi k}+\cos\theta\frac{\partial}{\partial\nu}\left(\frac{\mathrm{E}}{4\pi k}\right)+\cos^2\theta\frac{\partial^2}{\partial\nu^2}\left(\frac{\mathrm{E}}{4\pi k}\right)+\ldots \qquad (3)$$

The total radiant energy per unit volume, R, is

$$\begin{aligned}\mathrm{R}&=\frac{1}{\mathrm{C}}\int\int \mathrm{I}\sin\theta d\theta d\phi\\ &=\frac{1}{\mathrm{C}}\left[\frac{\mathrm{E}}{k}+\frac{1}{3}\frac{\partial^2}{\partial\nu^2}\left(\frac{\mathrm{E}}{k}\right)+\frac{1}{5}\frac{\partial^4}{\partial\nu^4}\left(\frac{\mathrm{E}}{k}\right)+\ldots\right] \qquad (4)\end{aligned}$$

and the total absorption per unit volume is $k\rho\mathrm{CR}$.

The net flow of radiation per unit time across a unit area perpendicular to Oz is

$$\begin{aligned}\mathrm{H}&=\int\int \mathrm{I}\cos\theta\sin\theta d\theta d\phi\\ &=\frac{1}{3}\frac{\partial}{\partial\nu}\left(\frac{\mathrm{E}}{k}\right)+\frac{1}{5}\frac{\partial^3}{\partial\nu^3}\left(\frac{\mathrm{E}}{k}\right)+\frac{1}{7}\frac{\partial^5}{\partial\nu^5}\left(\frac{\mathrm{E}}{k}\right)+\ldots \qquad (5)\end{aligned}$$

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* *M.N.*, **86** (1926), 379.

† Since writing this paper, my attention has been drawn to an important paper by Milne (*Proc. Camb. Phil. Soc.*, **21** (1923), 701), which I regret I had overlooked. In this Professor Milne obtains equations which are identical with those of my sect. 4, although (as the present paper shows) I am not in complete agreement with his discussion of the value to be assigned physically to $\mathrm{E}/4\pi k$ (his J). In view of this latter circumstance, I have thought it best to let my paper stand as originally written, while gladly conceding all priority in the equations of my sect. 4 to Professor Milne.

Across a unit area whose normal is in the direction Θ, Φ, the flow per unit time is

$$\int\int \mathrm{I}\{\cos\Theta\cos\theta+\sin\Theta\sin\theta\cos(\Phi-\phi)\}\sin\theta d\theta d\phi=\mathrm{H}\cos\Theta.$$

If p_{R} is the pressure of radiation in the same direction,

$$\frac{\partial p_{\mathrm{R}}}{\partial(z\sec\Theta)}=\frac{k\rho}{\mathrm{C}}\mathrm{H}\cos\Theta$$

so that

$$p_{\mathrm{R}}=\frac{1}{\mathrm{C}}\int \mathrm{H}d\nu$$

$$=\frac{1}{\mathrm{C}}\left[\frac{1}{3}\left(\frac{\mathrm{E}}{k}\right)+\frac{1}{5}\frac{\partial^2}{\partial\nu^2}\left(\frac{\mathrm{E}}{k}\right)+\frac{1}{7}\frac{\partial^4}{\partial\nu^4}\left(\frac{\mathrm{E}}{k}\right)+\ldots\right] \quad . \quad . \quad (6)$$

All these equations are exact in regions which are so far from the surface of the star that the term $Ae^{\nu\sec\theta}$ on the right of equation (2) may be neglected. We have, however, neglected the curvature of the star; if we wish to take this into consideration we must replace $\partial^2/\partial\nu^2$ by the substitution

$$\frac{\partial^2}{\partial\nu^2}=\sum_{x,\,y,\,z}\frac{1}{k\rho}\frac{\partial}{\partial x}\left(\frac{1}{k\rho}\frac{\partial}{\partial x}\right).$$

5. If the stellar material generates energy at the rate of $4\pi\epsilon$ per unit mass at the point under consideration, the equation of energy, expressing that the energy emitted per unit mass is the sum of the energy generated and the energy absorbed, is

$$\mathrm{E}=4\pi\epsilon+k\mathrm{CR} \quad . \quad . \quad . \quad . \quad (7)$$

With the value for CR obtained in equation (4) this becomes

$$\frac{4\pi\epsilon}{k}=-\frac{1}{3}\frac{\partial^2}{\partial\nu^2}\left(\frac{\mathrm{E}}{k}\right)-\frac{1}{5}\frac{\partial^4}{\partial\nu^4}\left(\frac{\mathrm{E}}{k}\right)-\frac{1}{7}\frac{\partial^6}{\partial\nu^6}\left(\frac{\mathrm{E}}{k}\right)-\ldots$$

Treating this as an equation for $\frac{\partial^2}{\partial\nu^2}\left(\frac{\mathrm{E}}{k}\right)$ and inverting, we find

$$\frac{\partial^2}{\partial\nu^2}\left(\frac{\mathrm{E}}{k}\right)=\left(-3+\frac{9}{5}\frac{\partial^2}{\partial\nu^2}+\frac{36}{175}\frac{\partial^4}{\partial\nu^4}+\ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad . \quad . \quad (8)$$

which shows how the emission of energy E, and hence also the temperature, depend on the distribution of ϵ.

On inserting this value, the equations of sect. 4 become

$$\mathrm{I}=\frac{\mathrm{E}}{4\pi k}+\cos\theta\frac{\partial}{\partial\nu}\left(\frac{\mathrm{E}}{4\pi k}\right)$$

$$+\left(\cos^2\theta+\cos^3\theta\frac{\partial}{\partial\nu}+\cos^4\theta\frac{\partial^2}{\partial\nu^2}+\ldots\right)\left(-3+\frac{9}{5}\frac{\partial^2}{\partial\nu^2}+\ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad (9)$$

$$\mathrm{R}=\frac{\mathrm{E}}{\mathrm{C}k}+\left(\frac{1}{3\mathrm{C}}+\frac{1}{5\mathrm{C}}\frac{\partial^2}{\partial\nu^2}+\frac{1}{7\mathrm{C}}\frac{\partial^4}{\partial\nu^4}+\ldots\right)\left(-3+\frac{9}{5}\frac{\partial^2}{\partial\nu^2}+\ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad . \quad (10)$$

$$p_{\mathrm{R}}=\frac{\mathrm{E}}{3\mathrm{C}k}+\left(\frac{1}{5\mathrm{C}}+\frac{1}{7\mathrm{C}}\frac{\partial^2}{\partial\nu^2}+\frac{1}{9\mathrm{C}}\frac{\partial^4}{\partial\nu^4}+\ldots\right)\left(-3+\frac{9}{5}\frac{\partial^2}{\partial\nu^2}+\ldots\right)\left(\frac{4\pi\epsilon}{k}\right) \quad . \quad (11)$$

6. On putting $\epsilon=0$ we pass to the case originally considered by Schwarzschild. The formula for I now reduces to its first two terms and so becomes identical with that given by Eddington. In general the error in Eddington's formula is represented by all the terms in $\cos^2\theta$, $\cos^3\theta$, etc., in formula (9), and these are of the order of ϵ, $\partial\epsilon/\partial\nu$, etc.

7. The usual definition of temperature presupposes the existence of a state of thermodynamical equilibrium. In problems such as the present in which thermodynamical equilibrium does not exist, there is no unique definition of the temperature and, just as in the corresponding problems in the theory of gases, we have a certain freedom of choice. This freedom of choice does not, of course, absolve us from the necessity of making our choice definitely and then adhering to it; we must avoid the possibility of introducing error by using the same symbol T to mean two different things.

Two obvious possibilities would be to define temperature in terms of the intensity of radiation by the equation

$$\mathrm{R}=a\mathrm{T}^4 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and to define it in terms of the emission of radiation by the equation

$$\mathrm{E}=ka\mathrm{C}\mathrm{T}^4 \quad . \quad . \quad . \quad . \quad (13)$$

The two definitions become identical when $\epsilon=0$, by equation (7), but when ϵ is not zero they are inconsistent, the difference between the two T's being of the order of ϵ.

There seems to be little advantage in definition (12) for, as the radiation is not distributed according to Planck's law, the T so defined would have no physical interest. Definition (13) has the advantage that the equations of sect. 4 for I, R, and p_{R} at once become equations in the single variable T, while T is expressed in terms of ϵ by equation (8). With this choice of definition for T the radiative equations take by far their simplest form.

In the equations of stellar equilibrium

$$d(p_{\mathrm{G}}+p_{\mathrm{R}})=\rho d[\mathrm{V}+\tfrac{1}{2}\omega^2(x^2+y^2)] \quad . \quad . \quad . \quad (14)$$

T enters both in p_{G} and p_{R}. Short of much more refined analysis than has so far been given, it does not seem possible to obtain the relation between the T which enters in p_{G}, which defines the mean kinetic energy of free electrons, etc., and the T which enters in p_{R}, no matter in what way we define the latter. The obvious plan of defining the second T to be the same as the first is foiled at the outset through the impossibility of determining p_{R} in terms of this T. The usual plan of taking p_{R} equal to $\frac{1}{3}a\mathrm{T}^4$ is seen, from equation (11), to involve an error of the order of ϵ. Until we know far more than at present about the mechanism of radiation, it appears impossible to obtain any dynamical equations which shall not contain errors of the order of ϵ.

8. The effect of the present paper has proved to be purely destructive. When sufficient care is taken of the small terms comparable with ϵ, a whole crop of theorems about stellar interiors are found to become untenable, and no corresponding new theorems have been found to take their places. The only positive result that has emerged is that all

theorems which make affirmations about quantities of the order of smallness of ϵ, and are based on the radiative equations of Eddington (or on any equivalent equations) must be scrapped. Particular attention may be directed to the theorems of Professor v. Zeipel, in which Professor Eddington has recently expressed his continued belief.*

The Equilibrium of the Calcium Chromosphere. (*Third Paper.*)
By Prof. E. A. Milne, F.R.S.

1. *Scope of the Paper.*—In previous papers † I have investigated the equilibrium of high-level atoms at rest in the solar or any stellar atmosphere under the influence of radiation-pressure and gravity. The present paper is concerned with the refinements necessary when we take into account the random velocities of the atoms. The main object of the paper is to ascertain the steady-state velocity-distribution of the high-level atoms. This leads to a consideration of points connected with the emission of prominences, and with the shift to the red of high-level lines found by St. John and others. The determination of the high-level velocity-distribution is in effect a determination of the temperature of the upper chromosphere as judged from the mean square velocity of the individual atoms.

2. *Consequences of the Doppler Effect.*—In a recent paper ‡ I have traced the effect of radiation-pressure on an outward-moving high-level atom which happens to become endowed with a velocity large enough to shift its characteristic absorption frequency into the violet wing of the absorption line formed by the atoms lower down. It was shown that since the residual intensity in the wing exceeds that in the centre of the line, the outward-moving atom becomes exposed, in consequence of the Doppler effect, to an increased radiation-pressure. By hypothesis, when it was at rest it was exposed to a radiation-pressure just sufficient to balance gravity. It is now, therefore, exposed to a radiation-pressure exceeding gravity. Hence it is subjected to an outward acceleration. This increases its velocity, and so sends it still further into the wing of the absorption line, where it is exposed to still more intense radiation, and therefore subjected to a still greater acceleration. The process goes on until the atom "climbs clean out of its own absorption line," or rather of the absorption line formed by the atoms below it. Meanwhile the acceleration is being reduced according to the inverse square law, and ultimately the atom moves outward with a definite, very large, limiting velocity.

It might at first sight appear that we have proved too much. If every atom which begins to move outwards is accelerated away in this way, how can the chromosphere have any permanent existence? It could apparently consist only of downward-moving atoms and atoms at

* *Nature*, 1926 May 1, Suppl., p. 28.
† *M.N.*, **84**, 354, 1924; **85**, 111, 1924; **86**, 8, 1925.
‡ *M.N.*, May 1926.

rest. But as shown in the paper mentioned, fast downward-moving atoms encounter an increased radiation-pressure in the red-ward wing of the absorption line, and this restores them to rest. However, it is absurd to suppose the chromosphere to consist entirely of atoms devoid of motion. Random fluctuation in the rate of absorption of energy would be expected to set up a pseudo-Maxwellian velocity-distribution corresponding to a temperature of the order of the effective temperature of the incident radiation. New outward-moving atoms would arise, which would apparently escape, and the chromosphere would be dissipated.

This effect must go on to some extent. At the same time, the chromosphere will not disappear. For for every atom that escapes, another atom will be ejected into the chromosphere from the lower layers, by radiation-pressure. The chromosphere acts like a cushion, the back radiation from which (together with its weight) is just sufficient to hold the atoms below in equilibrium against the radiation-pressure urging them outwards. If the cushion gives way, it is automatically re-created from below.

Nevertheless, when we come to inquire into the finer details of the equilibrium we see that it is only the faster outward-moving atoms that will be accelerated away. Consider an absorption line at $\nu=\nu_0$. An atom moving outwards with a *small* velocity v has its frequency shifted only slightly, and since the intensity I_ν has a minimum at or near $\nu=\nu_0$, $I_{\nu_0+\partial\nu}$ is practically equal to I_{ν_0}. Actually, to an observer moving with the atom, the intensity will appear slightly diminished, by a factor of the order of $1-v/c$, and so the atom will experience a radiation-pressure just less than will support it in equilibrium. It is therefore subject to a slight deceleration, proportional to v, and its velocity decreases, ultimately to zero. Slow outward-moving atoms thus come to rest, *on the average*, instead of being accelerated. Acceleration occurs only when the velocity v is so large that the "wing effect" overbalances the "v/c effect." Inward-moving atoms encounter an intensity slightly increased in the ratio $1+v/c$, and so quite apart from the "wing effect," which here acts in the same direction as the "v/c effect," these atoms also suffer a deceleration, and are reduced ultimately to rest.

We may summarise by saying that high-level equilibrium is stable for small velocity-changes.

3. *Pressure of Radiation on a Moving Atom.*—The details of the process are as follows: To make our ideas definite we will, to begin with, ignore the "wing effect," and suppose that the incident radiation is of intensity I, independent of ν. Consider a single atom in front of this source. The atom when at rest is capable of absorbing (and emitting) some definite frequency ν_0. In this frequency it is exposed to the intensity I. Now suppose the atom is moving away from the source of radiation with a velocity v, small compared with the velocity of light c. In consequence of its motion it will be exposed to a slightly less intensity, which differs from I by a fraction of the order of v/c. If it was previously just in equilibrium under radiation-pressure and gravity, gravity will now slightly exceed radiation-pressure, and the atom will experience an acceleration towards the source.

Crudely speaking, we may say that the atom loses in radiation-pressure because it is running away from the source of radiation. But that is a somewhat inadequate way of stating the matter—the phenomenon is really more complicated than that. What happens is described differently according as the observer moves with the atom or remains at rest with the source. To an observer moving with the atom, the atom continues to absorb and emit the same frequency ν_0. But the intensity of the oncoming radiation is weakened by a factor $(1-v/c)$; we may say that the quanta emitted by the source at the same rate as before reach the atom at longer intervals. Emissions, being random in direction, give no resultant momentum. The net result is a diminution in the rate of acquisition of momentum from the source. An observer at rest with respect to the source would describe things in an entirely different way, but he would reach the same conclusion. To him, the atom now requires quanta of higher frequency, $\nu_0(1+v/c)$. (This, the Doppler effect, arises because when the quanta arrive at the atom its motion reduces them by a factor $(1-v/c)$ back to its proper frequency ν_0.) Such quanta are larger, of energy $h\nu_0(1+v/c)$, and carry more momentum. But they arrive in smaller numbers, in the ratio $(1-v/c)$. These two effects therefore cut out. But every quantum absorbed has to be emitted before another can be absorbed, and to the observer at rest emissions are no longer random in direction. The atom appears to emit an excess of quanta in the direction of its motion. (This excess disappears of course when we transform back to the atom.) These quanta cause an excess of recoil momentum in the direction opposite to that of the atom's motion. The net result is a backward acceleration, *i.e.* one towards the source.

An exact calculation will be given later which will confirm the correctness of the foregoing verbal arguments.

In a similar manner an atom moving towards the source of radiation will be exposed to a slightly increased intensity, and will be subject to a net outward acceleration.

4. *Application to the Chromosphere.*—We apply this to the chromosphere. Here the source of radiation to which the high-level atoms are exposed is not uniform—it is given by the contour of the absorption line. Superposed on the effect just mentioned we have therefore the intrinsic change of intensity of the background for the Doppler displacement corresponding to v. For small velocities we may confine attention to the immediate neighbourhood of $\nu=\nu_0$, where the intensity-curve I_ν may be replaced by its tangent. For velocities sufficiently small for this to be legitimate, it is shown in § 6 below that the equation of motion for atoms moving with velocity v, assuming they are in equilibrium under radiation-pressure and gravity when $v=0$, is

$$\frac{1}{g}\frac{dv}{dt}=-\frac{2v}{c}\left[1-\tfrac{1}{3}\left(\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial \nu}\right)_{\nu=\nu_0}\right] \quad . \quad . \quad . \quad (1)$$

where g is the value of gravity. In (1) the darkening of the disc towards the limb is neglected; a more general formula, taking account of disc-darkening, is found below, but (1) is sufficient for our present purpose.

When v is larger, it is no longer justifiable to replace the intensity curve near ν_0 by its tangent. Roughly speaking, we have then to replace $\partial I_\nu/\partial\nu$ by $\Delta I/\Delta\nu$, where $\Delta\nu=(v/c)\nu_0$. When v is so large that $\nu_0+\Delta\nu$ lies in the violet wing, the term in $\Delta I/\Delta\nu$ is necessarily large and positive, and it is thus that the deceleration becomes changed into an acceleration when v is larger. For the present we continue to suppose v small.

The effect of " running away from the radiation " for an outward-moving atom, or of " running into the radiation " for an inward-moving atom, is thus to introduce a force analogous to a frictional resistance and proportional to v, for small values of v, provided always that

$$1>\tfrac{1}{3}\left(\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}\right)_{\nu=\nu_0} \quad . \quad . \quad . \quad . \quad (2)$$

It is readily seen that this condition is satisfied for an undisplaced line. For the intensity-distribution in any line is given by

$$I_\nu=r_\nu\frac{2h\nu^3/c^2}{e^{h\nu/kT}-1} \quad . \quad . \quad . \quad . \quad (3)$$

where T is the effective temperature of the photosphere and r_ν is the intensity-ratio. Neglecting a small term * in $e^{-h\nu/kT}$, we have

$$\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}=\frac{\nu}{r_\nu}\frac{dr_\nu}{d\nu}+3-\frac{h\nu}{kT},$$

whence

$$1-\frac{1}{3}\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}=\tfrac{1}{3}\left(\frac{h\nu}{kT}-\frac{\nu}{r_\nu}\frac{dr_\nu}{d\nu}\right) \quad . \quad . \quad . \quad (3')$$

For an undisplaced line, the absorption is a maximum at $\nu=\nu_0$, by hypothesis. Hence $(dr_\nu/d\nu)_{\nu=\nu_0}=0$. Thus (2) is satisfied.

5. *The Setting-up of a Steady State.*—The equation of motion (1) is not to be taken to imply that the velocity of every atom is gradually reduced to zero. It simply means that if we have a large number of atoms moving with velocity v, their average velocity is gradually reduced to zero, *i.e.* their centre of mass is gradually reduced to rest. The stabilising is with respect to the systematic motion. Whilst its centre of mass is coming to rest, the cloud of v-atoms will gradually become endowed with a random velocity-distribution of its own. This is due to the chance absorptions and emissions of quanta.

An analogous situation which arises in the kinetic theory of gases may make things clearer. Consider a large † volume of gas in equilibrium in a steady state. The molecules collide with one another, and set up a definite velocity-distribution, namely, that given by Maxwell's law. Take a specific direction Ox, and fix attention on all those molecules which at a particular instant are moving with a given velocity

* A similar approximation has been made in deducing (1). These small terms can readily be taken account of, but the formulæ are more complicated. The results are unchanged.

† We take the volume large so as to avoid the necessity of considering the effect of collisions with the walls.

v parallel to Ox. To these molecules, the velocity-distribution of the remaining molecules will be asymmetrical. In consequence of the net stream against them, their centre of mass will ultimately be reduced to rest, its equation of motion being of the form $dv/dt=-Av$. Meanwhile, they will suffer a dispersion in velocity which will result ultimately in being Maxwellian. From being a highly organised cloud, moving each with velocity v, they will be transformed into a cloud whose centre of gravity is at rest, the individual members of the cloud having velocities distributed in direction at random and in amount according to Maxwell's law.

The situation is just the same if the cloud of v-molecules is bombarded not by other molecules but by light quanta. We thus arrive at the conclusion that the high-level chromosphere possesses a permanent velocity-distribution.

It is the object of the following pages to calculate the permanent velocity-distribution so set up.

6. *Calculation of Radiation-Pressure.*—We proceed to calculate the radiation-pressure on an atom moving with velocity v, correct to the first order in v/c. In the first instance we know nothing as to the absorbing and radiating properties of an atom in motion. We derive such properties by changing to a frame of reference moving with the atom. In this frame the properties of the atom are those of an atom at rest. We require the formulæ for the Doppler effect and the aberration effect, and in addition a formula for the change in intensity of radiation from a moving source. The necessary formulæ are to be found in a famous paper of Einstein's.* Einstein in this paper does indeed calculate the pressure of radiation on a moving atom, but it is simpler to derive the expression *de novo* than to adapt his. Also Einstein's calculation had reference to atoms in thermodynamic equilibrium exposed to isotropic radiation, whilst our case is one of non-thermodynamic equilibrium in which the atoms are exposed to radiation on one side only.

Let v be the velocity of the atom in the positive direction of the axis of x. Let $\mathrm{I}(\nu, \theta)d\nu$ be the intensity of radiation, of frequency ν, in the interval $d\nu$, in a direction making an angle θ with the positive direction of x. We consider, in the first place, the pencil contained within an elementary solid angle $d\omega$.

Let the same symbols accented (′) refer to the same quantities measured in a frame of reference moving with velocity v. The formulæ of transformation are

$$\nu'=\nu\left(1-\frac{v}{c}\cos\theta\right) \quad . \quad . \quad . \quad . \quad (4)$$

$$\cos\theta'=\cos\theta-\frac{v}{c}+\frac{v}{c}\cos^2\theta \quad . \quad . \quad . \quad (5)$$

$$\frac{\mathrm{I}'(\nu', \theta')d\nu'd\omega'}{\mathrm{I}(\nu, \theta)d\nu d\omega}=1-2\frac{v}{c}\cos\theta \quad . \quad . \quad . \quad (6)$$

* *Phys. Zeit.*, **18**, 121, 1917.

Let ν_0 be the frequency absorbed and emitted by the atom when at rest. Then, as far as radiation capable of being absorbed by the atom is concerned, we have always $\nu'=\nu_0$.

The atom is alternately in its normal state (state 1) and in its excited state (state 2), and we have the usual relations between the probability coefficients.

$$\frac{A_{2\to1}}{B_{1\to2}}=\frac{2h\nu_0^3}{c^2}\cdot\frac{q_1}{q_2}, \qquad \frac{B_{2\to1}}{B_{1\to2}}=\frac{q_1}{q_2} \quad . \quad . \quad (7)$$

q_1 and q_2 being the statistical weights. The times spent by the atom in states 1 and 2 will fluctuate in accordance with the laws of probability. Instead, however, of calculating these directly, it seems simplest to consider a cloud of similar atoms of which at any instant n_1 are in state 1 and n_2 in state 2. The statement that a single atom makes as many transitions $1\to2$ in any interval as transitions $2\to1$ is then equivalent to the statement that the cloud of atoms is in a steady state.

The number of absorptions per second, counting stimulated emissions as negative absorptions in the usual way, is

$$\left(n_1-n_2\frac{q_1}{q_2}\right)B_{1\to2}\int I'(\nu',\theta')\frac{d\omega'}{4\pi},$$

which, by formulæ (4), (5), and (6), reduces to

$$\left(n_1-n_2\frac{q_1}{q_2}\right)B_{1\to2}\int I\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\ \theta\right)\left(1-\frac{v}{c}\cos\theta\right)\frac{d\omega}{4\pi}.$$

The number of emissions is $n_2A_{2\to1}$. Hence

$$n_2A_{2\to1}=\left(n_1-n_2\frac{q_1}{q_2}\right)B_{1\to2}\int I\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\ \theta\right)\left(1-\frac{v}{c}\cos\theta\right)\frac{d\omega}{4\pi} \qquad (8)$$

On introducing relations (7) we have the ratio n_2/n_1.

We now determine the rate of gain of momentum of the cloud of atoms. When a quantum $h\nu$, moving in direction θ, is absorbed, the x-component of momentum communicated is $(h\nu/c)\cos\theta$. The probability that an atom in state 1 does so absorb is $B_{1\to2}I'(\nu',\theta')d\omega'/4\pi$. Hence the gain of momentum per second (taking account of recoil momentum from stimulated emissions) is

$$\left(n_1-n_2\frac{q_1}{q_2}\right)B_{1\to2}\int I'(\nu',\theta')\frac{h\nu}{c}\cos\theta\frac{d\omega'}{4\pi}$$
$$=\left(n_1-n_2\frac{q_1}{q_2}\right)B_{1\to2}\int I\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\ \theta\right)\frac{h\nu_0}{c}\cos\theta\frac{d\omega}{4\pi} \quad . \qquad (9)$$

It should be noted that the factor $1-(v/c)\cos\theta$ coming in through substituting for I′ in terms of I is cancelled by the factor $1+(v/c)\cos\theta$ introduced by replacing ν by its expression in terms of ν_0. The "dilution effect" of running away from the source of radiation cuts out the Doppler effect.

It remains to calculate the recoil momentum arising from the

unstimulated emissions. For an atom at rest these give zero momentum on the average, since emissions are distributed at random in direction. The same is therefore true for the moving atom, *for the momentum calculated with reference to the moving frame of reference.* But the momentum calculated with reference to the frame at rest, which is what we are interested in, is not zero. This may appear paradoxical, and we shall therefore give two demonstrations.

The chance of an emission inside $d\omega'$ in one second is simply $A_{2\to1}d\omega'/4\pi$. Hence the loss of momentum per second is

$$n_2A_{2\to1}\int\frac{h\nu}{c}\cos\theta\frac{d\omega'}{4\pi}.$$

We use $(h\nu/c)\cos\theta$, for this is the measure of the change in momentum-component measured in the fixed frame. Now

$$\frac{d\omega'}{d\omega}=\frac{d(\cos\theta')}{d(\cos\theta)}=1+2\frac{v}{c}\cos\theta.$$

Hence, transforming, the loss of momentum is

$$n_2A_{2\to1}\frac{h\nu_0}{c}\int\left(1+\frac{v}{c}\cos\theta\right)\cos\theta\left(1+2\frac{v}{c}\cos\theta\right)\frac{d\omega}{4\pi},$$

where the integral is taken over the complete solid angle. To the first order in v/c this is found to be

$$n_2A_{2\to1}\frac{h\nu_0}{c}\cdot\frac{v}{c} \quad . \quad . \quad . \quad . \quad (10)$$

We see that this vanishes when $v=0$, as it should. This method of calculation corresponds to the circumstance that the emissions, though random in direction to a moving observer, are not so to a fixed observer. To a fixed observer there is an excess of emissions in the direction of motion, resulting in a net loss of momentum.

The alternative deduction is as follows :—When an atom emits and loses energy $h\nu_0$, it also loses mass of amount δm, given by $\delta m=h\nu_0/c^2$. If it is moving with velocity v, this loss of mass involves loss of momentum, of amount $v\delta m=vh\nu_0/c^2$. Multiplying by $n_2A_{2\to1}$ for the number of such emissions, we recover formula (10).

Let f be the acceleration produced by radiation-pressure. If m is the mass of the atom $(n_1+n_2)mf$ is equal to the excess of (9) over (10). Using (8) to eliminate $n_2A_{2\to1}$, we find

$$mf=\frac{n_1-n_2(q_1/q_2)}{n_1+n_2}\frac{h\nu_0}{c}B_{1\to2}\left[\int I\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\theta\right)\cos\theta\frac{d\omega}{4\pi}\right.$$
$$\left.-\frac{v}{c}\int I\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\theta\right)\frac{d\omega}{4\pi}\right] \quad . \quad (11)$$

In this we have dropped higher powers of v/c than the first, except that we have retained the exact value of the argument of I in the second term ; the reason for this is that in an absorption line I may change very rapidly with ν. The ratio n_2/n_1 still appears explicitly in (11), but

it can be eliminated by a further use of (8) and (7) if an exact expression is required. However, n_2/n_1 is usually so small that we can replace

$$\frac{n_1 - n_2(q_1/q_2)}{n_1+n_2}$$

by unity without serious error in most applications;* it contains, of course, a term in v/c, but as this only appears as a correction to the departure from unity we can ignore it. The integrals in (11) are to be taken over the whole solid angle in which there is any incident radiation.

We now apply the result to a high-level atom acted on by radiation-pressure and gravity. The equation of motion is

$$\frac{dv}{dt} = -g+f \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If the intensity for frequency ν_0 is such that the atom is just in equilibrium when $v=0$, we have

$$0 = -g+f_{v=0} \quad . \quad . \quad . \quad . \quad (13)$$

Hence, eliminating $B_{1\to 2}$, we find

$$\frac{1}{g}\frac{dv}{dt} = -1+\frac{f}{f_{v=0}} = \frac{\int\left[\mathrm{I}\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\theta\right)-\mathrm{I}(\nu_0,\theta)\right]\cos\theta\frac{d\omega}{4\pi} - \frac{v}{c}\int \mathrm{I}\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\theta\right)\frac{d\omega}{4\pi}}{\int \mathrm{I}(\nu_0,\theta)\cos\theta\frac{d\omega}{4\pi}} \quad (14)$$

We now confine ourselves to very small values of v. In that case we may write

$$\mathrm{I}\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\theta\right)-\mathrm{I}(\nu_0,\theta) = \frac{v}{c}\nu_0\cos\theta\left[\frac{\partial \mathrm{I}(\nu,\theta)}{\partial\nu}\right]_0.$$

This is equivalent to adopting the tangent to the energy-curve at $\nu=\nu_0$ as giving the contour near ν_0 sufficiently accurately. Then (14) becomes

$$\frac{1}{g}\frac{dv}{dt} = \frac{v}{c}\cdot\frac{\nu_0\int\left[\frac{\partial \mathrm{I}(\nu,\theta)}{\partial\nu}\right]_0\cos^2\theta\frac{d\omega}{4\pi} - \int \mathrm{I}(\nu_0,\theta)\frac{d\omega}{4\pi}}{\int \mathrm{I}(\nu_0,\theta)\cos\theta\frac{d\omega}{4\pi}} \quad . \quad (15)$$

Suppose now that the coefficient of darkening of the sun's disc in frequency ν is α_ν. Then

$$\mathrm{I}(\nu,\theta) = \mathrm{I}(\nu,0)(1-\alpha_\nu+\alpha_\nu\cos\theta)$$

and

$$\frac{\partial \mathrm{I}(\nu,\theta)}{\partial\nu} = \frac{\partial \mathrm{I}(\nu,0)}{\partial\nu}(1-\alpha_\nu+\alpha_\nu\cos\theta) - \mathrm{I}(\nu,0)\frac{d\alpha_\nu}{d\nu}(1-\cos\theta).$$

* Not in all. The complete density-distribution in the chromosphere depends on the fact that this factor is not exactly unity, as has been shown in previous papers.

We find then

$$\frac{1}{g}\frac{dv}{dt}=\frac{v}{c}\left[\frac{\frac{1}{3}(1-\frac{1}{4}a_\nu)\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}-1\frac{1}{2}\nu\frac{da_\nu}{d\nu}-(1-\frac{1}{2}a_\nu)}{\frac{1}{2}(1-\frac{1}{3}a_\nu)}\right]_{\nu=\nu_0} \quad . \quad (16)$$

where I_ν denotes $I(\nu, o)$.

Particular cases of this are of interest. Neglecting the small term $\frac{1}{1\frac{1}{2}}\nu da_\nu/d\nu$, we have

$$\text{for} \quad a_\nu=o, \quad \frac{1}{g}\frac{dv}{dt}=-2\frac{v}{c}\left(1-\frac{1}{3}\frac{\nu}{I}\frac{\partial I}{\partial\nu}\right)_0 \quad . \quad . \quad . \quad (17)$$

$$\text{for} \quad a_\nu=\tfrac{3}{5}, \quad \frac{1}{g}\frac{dv}{dt}=-\frac{v}{c}\left(\frac{7}{4}-\frac{17}{24}\frac{\nu}{I}\frac{\partial I}{\partial\nu}\right)_0 \quad . \quad . \quad . \quad (17')$$

$$\text{for} \quad a_\nu=1, \quad \frac{1}{g}\frac{dv}{dt}=-\frac{v}{c}\left(\frac{3}{2}-\frac{3}{4}\frac{\nu}{I}\frac{\partial I}{\partial\nu}\right)_0 \quad . \quad . \quad . \quad (17'')$$

For $I_\nu=r_\nu(2h\nu^3/c^2)(e^{h\nu/k}-1)^{-1}$, with $dr_\nu/d\nu=o$ for $\nu=\nu_0$, these become approximately

$$\text{for} \quad a_\nu=o, \quad \frac{1}{g}\frac{dv}{dt}=-\frac{2}{3}\frac{v}{c}\frac{h\nu_0}{kT} \quad . \quad . \quad . \quad . \quad (18)$$

$$\text{for} \quad a_\nu=\tfrac{3}{5}, \quad \frac{1}{g}\frac{dv}{dt}=-\frac{v}{c}\left(\frac{17}{24}\frac{h\nu_0}{kT}-\frac{3}{8}\right) \quad . \quad . \quad . \quad (18')$$

$$\text{for} \quad a_\nu=1, \quad \frac{1}{g}\frac{dv}{dt}=-\frac{v}{c}\left(\frac{3}{4}\frac{h\nu_0}{kT}-\frac{3}{4}\right) \quad . \quad . \quad . \quad (18'')$$

It may be recalled that for the H and K lines on the sun, $h\nu_0/kT$ is about 6. The coefficients of $-v/c$ in the three cases are then 4, $\frac{9}{2}\frac{5}{4}$, $\frac{15}{4}$ respectively.

7. *The Velocity-Distribution.*—We proceed to determine the atomic velocity-distribution in the high-level chromosphere in the steady state. We follow a method, first used by Einstein in his investigations on Brownian motion and afterwards used by him to calculate the steady-state velocity-distribution in a gas consisting of emitting and absorbing atoms. Our problem differs from Einstein's in that the radiation is not isotropic, and that there is an external force acting, namely, gravity.

Write the equation of motion (16) in the form

$$\frac{dv}{dt}=-\lambda v.$$

After a time t, the average velocity of a group of v-atoms is reduced to $ve^{-\lambda t}$. Meanwhile they have been acquiring individual random velocities. Let u be the random velocity acquired, in the x-direction, in time t by any particular atom. Then the velocity of this atom in the x-direction is now $ve^{-\lambda t}+u$. A necessary condition for a steady state is

$$\overline{(ve^{-\lambda t}+u)^2}=\overline{v^2},$$

where the bar denotes the mean value taken over all the atoms. Remembering $\bar{u}=0$, we find

$$\overline{u^2}=\overline{v^2}(1-e^{-2\lambda t}) \quad . \quad . \quad . \quad . \quad (19)$$

For t small, this becomes

$$\overline{v^2}=\frac{\overline{u^2}}{2\lambda t} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which is Einstein's result.

We now calculate $\overline{u^2}$. Since we have already taken account of the systematic influence of the radiation on the group of v-atoms, arising from their motion, we can ignore the motion in calculating $\overline{u^2}$.

Suppose there are N absorptions per atom, on the average, in the time-interval t. Then there will be N emissions. Let N′ of these be stimulated emissions, N″ unstimulated. Then $N=N'+N''$, and we have

$$\frac{(n_1+n_2)N}{t}=n_1B_{1\to 2}\int I(\nu,\theta)\frac{d\omega}{4\pi}=\tfrac{1}{2}n_1B_{1\to 2}I(\nu,0)(1-\tfrac{1}{2}\alpha_\nu),$$

$$\frac{(n_1+n_2)N'}{t}=n_2\frac{q_1}{q_2}B_{1\to 2}\int I(\nu,\theta)\frac{d\omega}{4\pi}=\tfrac{1}{2}n_2\frac{q_1}{q_2}B_{1\to 2}I(\nu,0)(1-\tfrac{1}{2}\alpha_\nu),$$

$$\frac{(n_1+n_2)N''}{t}=n_2A_{2\to 1}.$$

Let the sth absorption be that of a quantum moving in direction θ_s; let the sth stimulated emission be that of a quantum in direction θ_s'; and let the sth unstimulated emission be that of a quantum in direction θ_s''. Then the increase of velocity u in the interval t is given by

$$u=-gt+\frac{h\nu_0}{mc}\Big[\sum_1^N\cos\theta_s-\sum_1^{N'}\cos\theta_s'-\sum_1^{N''}\cos\theta_s''\Big].$$

The condition for a state of equilibrium is

$$0=-gt+\frac{h\nu_0}{mc}[N\,\overline{\cos\theta}-N'\,\overline{\cos\theta'}] \quad . \quad . \quad . \quad (21)$$

since $\overline{\cos\theta''}=0$, unstimulated emissions being random in direction. In the expressions $\overline{\cos\theta}$, $\overline{\cos\theta'}$, the bar denotes the mean value averaged over the possible values of θ that occur, *i.e.* over the solid angle $\phi=0$ to 2π, $\theta=0$ to $\frac{1}{2}\pi$. Relation (21) is of course equivalent to (13). We have then

$$u=\frac{h\nu_0}{mc}\Big[\sum_1^N(\cos\theta_s-\overline{\cos\theta})-\sum_1^{N'}(\cos\theta_s'-\overline{\cos\theta'})-\sum_1^{N''}\cos\theta_s''\Big].$$

The three terms in the square bracket have each a mean value zero, and are uncorrelated with one another. Hence

$$\overline{u^2}=\Big(\frac{h\nu_0}{mc}\Big)^2\Big[N\,\overline{(\cos\theta_s-\overline{\cos\theta})^2}+N'\,\overline{(\cos\theta_s-\overline{\cos\theta'})^2}+N''\,\overline{\cos^2\theta''}\Big] \quad (22)$$

Now

$$\overline{(\cos\theta_s-\overline{\cos\theta})^2}=\overline{\cos^2\theta}-(\overline{\cos\theta})^2,$$

and we have

$$\overline{\cos^2\theta}=\overline{\cos^2\theta'}=\frac{\int I(\nu,\,\theta)\cos^2\theta\frac{d\omega}{4\pi}}{\int I(\nu,\,\theta)\frac{d\omega}{4\pi}}=\frac{\frac{1}{3}(1-\frac{1}{4}\alpha_\nu)}{1-\frac{1}{2}\alpha_\nu},$$

$$\overline{\cos\theta}=\overline{\cos\theta'}=\frac{\int I(\nu,\,\theta)\cos\theta\frac{d\omega}{4\pi}}{\int I(\nu,\,\theta)\frac{d\omega}{4\pi}}=\frac{\frac{1}{2}(1-\frac{1}{3}\alpha_\nu)}{1-\frac{1}{2}\alpha_\nu}.$$

Further,

$$\overline{\cos^2\theta''}=\tfrac{1}{3}.$$

Hence

$$\overline{u^2}=\tfrac{1}{2}\frac{B_{1\to2}I(\nu,\,0)(1-\frac{1}{2}\alpha_\nu)}{n_1+n_2}\left(\frac{h\nu_0}{mc}\right)^2 t$$
$$\left[\left(n_1+n_2\frac{q_1}{q_2}\right)\left\{\frac{\frac{1}{3}(1-\frac{1}{4}\alpha_\nu)}{1-\frac{1}{2}\alpha_\nu}-\frac{\frac{1}{4}(1-\frac{1}{3}\alpha_\nu)^2}{(1-\frac{1}{2}\alpha_\nu)^2}\right\}+\tfrac{1}{3}\left(n_1-n_2\frac{q_1}{q_2}\right)\right].$$

But (21) gives

$$g=\frac{h\nu_0}{mc}\frac{n_1-n_2(q_1/q_2)}{n_1+n_2}\tfrac{1}{2}B_{1\to2}I(\nu,\,0)\tfrac{1}{2}(1-\tfrac{1}{3}\alpha_\nu) \quad . \quad . \quad (23)$$

Hence we find for $\overline{u^2}$ the expression

$$\overline{u^2}=gt\frac{h\nu_0}{mc}\frac{1-\frac{1}{2}\alpha_\nu}{\frac{1}{2}(1-\frac{1}{3}\alpha_\nu)}\left[\frac{n_1+n_2(q_1/q_2)}{n_1-n_2(q_1/q_2)}\cdot\frac{\frac{1}{1\cdot2}(1-\alpha_\nu+\frac{1}{6}\alpha_\nu^2)}{(1-\frac{1}{2}\alpha_\nu)^2}+\tfrac{1}{3}\right] \quad . \quad (24)$$

In this the factor in n_2/n_1 may be replaced by unity.

From (16) we have

$$\lambda=\frac{g}{c}\left[\frac{(1-\frac{1}{2}\alpha_\nu)+\frac{1}{1\cdot2}\nu\frac{d\alpha_\nu}{d\nu}-\frac{1}{3}(1-\frac{1}{4}\alpha_\nu)\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}}{\frac{1}{2}(1-\frac{1}{3}\alpha_\nu)}\right]_{\nu=\nu_0} \quad . \quad (25)$$

Hence, finally, for the mean square velocity in the steady state

$$\overline{v^2}=\frac{\overline{u^2}}{2\lambda t}=\frac{1}{2}\frac{h\nu_0}{m}(1-\tfrac{1}{2}\alpha_\nu)_0\left[\frac{\frac{\frac{1}{1\cdot2}(1-\alpha_\nu+\frac{1}{6}\alpha_\nu^2)}{(1-\frac{1}{2}\alpha_\nu)^2}+\frac{1}{3}}{1-\frac{1}{2}\alpha_\nu+\frac{1}{1\cdot2}\nu\frac{\partial\alpha_\nu}{\partial\nu}-\frac{1}{3}(1-\frac{1}{4}\alpha_\nu)\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}}\right]_{\nu=\nu_0} \quad . \quad (26)$$

In this, I_ν stands for $I(\nu,\,0)$.

Before evaluating $\overline{v^2}$ for the chromosphere by this formula, the following simple check may be applied. If one casts a glance back through the analysis, one sees that the term $\frac{1}{1\cdot2}(1-\alpha_\nu+\frac{1}{6}\alpha_\nu^2)(1-\frac{1}{2}\alpha_\nu)^{-2}$ arises from the fact of the incident radiation having to balance gravity. If gravity were not acting and the radiation were isotropic, the term just mentioned would be equal to $\frac{1}{3}$, the value of $\overline{\cos^2\theta''}$. We should then have the case of the thermodynamic equilibrium of a mass of gas, in the presence of radiation, under no external forces. Calling

the mean square velocity in this case $\overline{V^2}$ and putting $\alpha_\nu=0$, we should have

$$\overline{V^2}=\frac{\frac{1}{3}h\nu_0/m}{1-\frac{1}{3}\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}}.$$

But now I_ν necessarily denotes black radiation of temperature T. Thus $I_\nu=(2h\nu^3/c^2)(e^{h\nu/kT}-1)^{-1}$, and approximately

$$1-\frac{1}{3}\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}=\frac{1}{3}\frac{h\nu_0}{kT}.$$

Hence

$$\overline{V^2}=kT/m,$$

or

$$\tfrac{1}{2}m\overline{V^2}=\tfrac{1}{2}kT.$$

Remembering that V is the component velocity in a specific direction we see that $\frac{1}{2}m\overline{V^2}$ has precisely the value given by general statistical mechanics, as, of course, it should have if the theory and analysis are correct. This was, in fact, the way Einstein proved that his theory of radiation, with directed light quanta possessing directed momenta both in absorption and in emission, was consistent with statistical mechanics.

Three particular cases of (26) are of special interest. They are: the uniform disc ($\alpha_\nu=0$), the fully darkened disc ($\alpha_\nu=1$), and the distribution $\alpha_\nu=\frac{3}{5}$. The latter I have shown to be not only the coefficient of darkening for integrated radiation, but also the coefficient of darkening for the residual intensity in a line formed by atoms in monochromatic radiative equilibrium, as in the high-level chromosphere. For simplicity we shall ignore the small term $\frac{1}{12}\nu d\alpha_\nu/d\nu$. We thus find

$$\text{for}\quad \alpha_\nu=0,\qquad \overline{v^2}=\frac{h\nu_0}{m}\cdot\frac{\frac{5}{8}}{3-\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}}\qquad\ldots\qquad(27)$$

$$\text{for}\quad \alpha_\nu=\tfrac{3}{5},\qquad \overline{v^2}=\frac{h\nu_0}{m}\cdot\frac{\frac{3}{7}\frac{6}{14}\frac{3}{}}{\frac{4}{14}\frac{2}{7}-\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}}\qquad\ldots\qquad(27')$$

$$\text{for}\quad \alpha_\nu=1,\qquad \overline{v^2}=\frac{h\nu_0}{m}\cdot\frac{\frac{7}{18}}{2-\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}}\qquad\ldots\qquad(27'')$$

We now put

$$I_\nu=r_\nu\frac{2h\nu^3/c^2}{e^{h\nu/kT_1}-1},$$

where r_ν is the intensity ratio in the line at the centre of the disc and T_1 is the effective temperature at the centre of the disc. For $\nu=\nu_0$, the ratio-reduction in intensity is a maximum, and $dr_\nu/d\nu=0$. Hence

$$\left(\frac{\nu}{I_\nu}\frac{\partial I_\nu}{\partial\nu}\right)_0=3-\frac{h\nu}{kT}.$$

Hence we have

$$\text{for } \alpha_\nu=0, \quad \overline{v^2}=\frac{5}{8}\frac{k}{m}T_1 \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\text{for } \alpha_\nu=\tfrac{3}{5}, \quad \overline{v^2}=\tfrac{3}{7}\tfrac{6}{1}\tfrac{3}{4}\left(1-\frac{9}{17}\frac{kT_1}{h\nu_0}\right)^{-1}\frac{k}{m}T_1 \quad . \quad . \quad (28')$$

$$\text{for } \alpha_\nu=1, \quad \overline{v^2}=\tfrac{7}{18}\left(1-\frac{kT_1}{h\nu_0}\right)^{-1}\frac{k}{m}T_1 \quad . \quad . \quad . \quad (28'')$$

If now T_0 is the temperature of the upper chromosphere *as defined by the mean square outward velocity-component of the atoms*, we have

$$\tfrac{1}{2}m\overline{v^2}=\tfrac{1}{2}kT_0.$$

Hence,

$$\text{for } \alpha_\nu=0, \quad T_0=\tfrac{5}{8}T_1 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\text{for } \alpha_\nu=\tfrac{3}{5}, \quad T_0=\tfrac{3}{7}\tfrac{6}{1}\tfrac{3}{4}\left(1-\frac{9}{17}\frac{kT_1}{h\nu_0}\right)^{-1}T_1 \quad . \quad . \quad (29')$$

$$\text{for } \alpha_\nu=1, \quad T_0=\tfrac{7}{18}\left(1-\frac{kT_1}{h\nu_0}\right)^{-1}T_1 \quad . \quad . \quad . \quad (29'')$$

For the sun, the fraction $kT_1/h\nu_0$ is about $\frac{1}{6}$ for the H and K lines of Ca^+. We have therefore $T_0=0{\cdot}63T_1, T_0=0{\cdot}56T_1, T_0=0{\cdot}47T_1$, respectively. The value of $(\overline{v^2})^{\frac{1}{2}}$ from (28) for a calcium atom with $T_1=6000°$ is about 0·36 km./sec.

The effective temperature T_e of the disc as a whole is given by

$$T_e{}^4=T_1{}^4(1-\tfrac{1}{3}\alpha_\nu),$$

where $\alpha_\nu=\frac{3}{5}$, and hence $T_e=0{\cdot}95T_1$. Hence, surveying our results as a whole, we may state our main conclusion as follows : *the temperature of the high-level chromosphere as judged from the mean square outward velocity-component of its atoms is slightly more than one-half the effective temperature of the star as a whole.*

So far we have only determined $\overline{v^2}$ and the temperature to which it corresponds. Are we justified in inferring that the velocity-distribution is the Maxwellian distribution for this temperature ? In a paper communicated by the author to the *Proceedings* of the Cambridge Philosophical Society, it is shown formally that if the motion of each atom is a deceleration of the type $dv/dt=-\lambda v$, superposed on a random acquisition of velocity, then the steady state set up is characterised by the velocity-distribution

$$e^{-\frac{1}{2}v^2/\overline{v^2}}dv,$$

so that if $\frac{1}{2}\overline{mv^2}=\frac{1}{2}kT_0$, the distribution is

$$e^{-\frac{1}{2}mv^2/kT_0}dv,$$

i.e. the Maxwellian distribution for the velocity-component v at temperature T_0.

The state of the gas must be carefully distinguished from one of

thermodynamic equilibrium at temperature T_0. T_0 is merely a parameter describing the mean square velocity. Any other property might be taken and used to define an equivalent temperature, but it will not, in general, give the same value T_0. For example, I have shown in a previous paper that the ratio n_2/n_1 of atoms in the excited and normal states is given by

$$\frac{n_2(q_1/q_2)}{n_1-n_2(q_1/q_2)}=\frac{\frac{1}{2}r_\nu}{e^{h\nu/kT_1}-1}.$$

The equivalent temperature T′ (*i.e.* the temperature at which n_2/n_1 would have the same value in thermodynamic equilibrium) is given by

$$n_2/n_1=(q_2/q_1)e^{-h\nu/kT'},$$

whence

$$\frac{1}{e^{h\nu/kT'}-1}=\frac{\frac{1}{2}r_\nu}{e^{h\nu/kT_1}-1}.$$

Solving numerically for the H and K lines with $T_1=6000°$, we find $T'=4060°$. On the other hand, for the velocity-distribution we have $T_0=\frac{5}{8}T_1=3750°$. The difference is a measure of the departure from a state of thermodynamic equilibrium. It is interesting to note that the one estimate (velocity-distribution) depends only on T_1 and is independent of r_ν; the other (ratio in excited and normal states) depends on r_ν, and therefore on the atomic constant τ.

It seems to me that it is worth dwelling on the result that the velocity-distribution is Maxwellian. We are familiar with Maxwellian distributions in thermodynamic equilibrium, and even an external field, under these circumstances, leaves the distribution a Maxwellian one. But in the present case there is in addition a one-sided field of radiation balancing the external gravitational field; the system is not enclosed, the radiation passes through and escapes. Yet the velocity-distribution is Maxwellian. It appears to me that this is a state of affairs new to mathematical physics.

Applications.

8. *The Motion of Prominences.*—In an earlier paper, discussing the possibility of the occasional collapse of the chromosphere and its reformation from below, I remarked * that "the abrupt changes of form, position, and velocity shown by prominences are doubtless essentially the same phenomenon on a catastrophic scale, arising from sudden local increases or diminutions in the corresponding ν-radiation." It is clear, for example, that if the residual intensity in H and K for the undisturbed sun just supports a Ca^+ atom against gravity, then if the residual intensity is doubled the atom will be driven outwards under an initial acceleration equal to gravity. Now in the region of H and K an increase in temperature from 6000° to 6800° doubles the intensity. Thus comparatively small increases in effective temperature will lead to the expulsion of atoms under an acceleration.

* *M.N.*, **85**, 133, 1924.

If the enhanced temperature persists, a new steady state of equilibrium will ultimately be called into being, when a screen has been ejected thick enough to reduce the residual intensity to its original value. It may take the form of a complete chromosphere, or it may take that of a quiescent prominence if the rise of temperature is local. But *sudden* local increases of temperature will cause temporary ejections of atoms which we shall recognise as eruptive prominences.

Detailed calculations of this character, taking into account the possibility of increase of temperature over a limited area, have recently been made by R. K. Sur.* He does not, however, take into account the consequences of the Doppler effect in moving the atom's absorption frequency into the wing of the absorption line. The accelerations, he calculates, are therefore only initial accelerations ; unless the mass in motion is sufficiently large to carry the whole absorption line with it, much larger accelerations will be experienced for the same rise of temperature, as soon as an appreciable outward velocity has been acquired.

But he does refer to Einstein's theory of the change of radiation-pressure for an atom in motion. It appears to me that the conclusion Sur comes to on this point is erroneous, and as the point is precisely that which is the subject of the present paper, I propose to examine it in detail. At the end of his paper, Sur criticises some conclusions in a recently published memoir by Pettit.† This memoir contains the most complete account of the observed motions of prominences that is available, and I can only express my admiration of so magnificent a collection of data. At the same time, with Sur I am compelled to dissent from Pettit's theoretical conclusions.

Pettit considers that radiation-pressure is totally inadequate to account for the observed prominence-motions. But the formula he uses is that due to Schwarzschild, Nicholson, Proudman, and Debye, which determines the pressure of radiation on a spherical particle of radius small compared with the wave-length of the incident light. According to Pettit, it gives a pressure only 10^{-11} of that which would be exerted if all the radiation in an area equal to the particle's cross-section communicated its momentum to the atom. But in the derivation of this formula, the particle is assumed to possess the macroscopic properties of gross matter. Roughly speaking, almost all the radiation incident on such a particle is diffracted round it. But the actual properties of the atom are far removed from the properties of such a hypothetical particle. Both the observed and calculated values for the atomic absorption coefficient at a characteristic frequency show this coefficient to be of the same order as the actual cross-section of the atom, *i.e.* about 10^{11} times as big as determined by Pettit. Pettit's memoir, though published in 1925, is dated 1920, at which time the extraordinary opacity of an atom at an absorption frequency was scarcely realised. The quantum theory of atom structure puts the older estimates entirely out of court.

Pettit's other objections to the adequacy of radiation-pressure may be briefly mentioned. (1) He says that if radiation-pressure at the sun's surface exceeded gravity, neither the chromosphere nor quiescent

* *Astrophys. Journ.*, **63**, 111, 1926.
† *Pub. Yerkes Observatory*, **3**, Part 4, 205–239 (1925).

prominences could exist. On the contrary, it is precisely because radiation-pressure on certain atoms at the sun's surface exceeds gravity that the chromosphere exists at all. The chromosphere comes into being as a screen of atoms ejected by the sun until it is thick enough to cut down radiation-pressure until it equals gravity, *inside the absorption line so formed.* (2) and (3) He says that the velocities of eruptive prominences would be nearly the same at the same height, and would be radial to the sun. But, as Sur points out, the velocities depend on the amount of the local increase of temperature of the photosphere, and need only to be radial to the local centre of increased temperature.

Objection (4) brings us to the subject of the present paper. Pettit's curves of height against time for the motions of individual prominences are analysed by him into segments of straight lines, which imply that the motion consists of a uniform velocity for a certain period, followed by an abruptly increased uniform velocity for another period, and so on. Pettit urges that if radiation-pressure produced the motion, prominences would show continuous acceleration rather than successive periods of uniform velocity.

In the first place, I am unable to convince myself, from a study of Pettit's graphs, that they are so incompatible with a motion of continuous acceleration as he states. In at least some of his diagrams, a parabolic-looking curve would fit the observations as well as a series of straight lines.

In the second place, abrupt changes of velocity followed by periods of uniform velocity are perfectly compatible with motion under radiation-pressure. We merely have to suppose that the photosphere brightens for a short time, and then relapses to its original brightness. The original brightness being just sufficient to balance gravity, we should have periods of zero acceleration separated by short periods of rapid acceleration.

Sur, however, accepts Pettit's analysis of the motions, and then argues as follows to account for the periods of uniform velocity. He points out that a particle subject to an accelerating force can acquire a constant velocity if it moves in a viscous medium, the equation of motion being (in his notation)

$$m\frac{dv}{dt}=-\mathrm{R}v+\mathrm{E}$$

and the terminal velocity being E/R. He then says that, according to Einstein's theory of radiation, a resonator moving in a field of radiation experiences a frictional force opposite to its motion and proportional to its velocity. Quoting a formula derived by Einstein for an atom moving in a field of isotropic black radiation of temperature T, he then says that with a suitable value of $B_{1\to 2}$ it appears that the frictional term is quite sufficient to reduce the motion to one of constant limiting velocity.

This conclusion is erroneous. It appears to me that Sur has been misled by taking Einstein's formula out of its context and applying it in circumstances in which it is not applicable. In the first place, it is obvious on general grounds that the motion of the atom can only

modify the pressure of radiation by a fraction * of the order of v/c. Now even a velocity of 300 km./sec. is only 1/1000th of that of light, so it is quite unreasonable to expect that the change in the acceleration due to v can amount to a quantity of the order of magnitude of the original acceleration. The term $-Rv$ is simply the correction to E due to v, and is of the order $E(v/c)$. But, in the second place, Einstein's formula requires modification when applied to such large velocities as 300 km./sec. *in the neighbourhood of an absorption line.*

At the end of § 6 above, I have in effect evaluated Sur's term $-Rv$ for velocities so small that the contour of the intensity-curve near $\nu=\nu_0$ may be supposed to coincide with its tangent. This assumption was also made by Einstein, but here it is much more restrictive. In Einstein's case the radiation was black—the curve is smooth. Near an absorption frequency, however, the contour is turning comparatively suddenly, and can be identified with the tangent at $\nu=\nu_0$ probably only over a region corresponding to the velocities of thermal agitation—the $\sqrt{\overline{v^2}}$ of § 7. At the end of § 6 it was found that the contribution to dv/dt for small velocities is about $-4g(v/c)$ for the H and K lines of Ca^+ on the sun. For this to equal, say, even $\frac{1}{10}\,g$, we should have to have $v/c=1/40$, or $v=7500$ km./sec. No such velocities are observed, of course, in prominences. But in any case a velocity such that $v/c=1/40$ would have brought in a Doppler shift of 100Å, taking the atoms clean away from the original part of the spectrum and violating the assumption about the intensity-contour.

When we take into account both the Doppler effect and the absorption-line contour, Einstein's method yields, as we have seen,†

$$\frac{1}{g}\frac{dv}{dt}=-\frac{2v}{c}\left(1-\frac{1}{3}\frac{\nu_0}{\mathrm{I}}\frac{\Delta \mathrm{I}}{\Delta\nu}\right) \quad . \quad . \quad . \quad (30)$$

where $\Delta\nu=(v/c)\nu_0$ and ΔI is the change in intensity for the displacement $\Delta\nu$. As soon as $v=300$ km./sec., $\Delta\nu$ is the equivalent of 4Å, and the quantity

$$\frac{1}{3}\frac{v}{c}\frac{\nu_0}{\mathrm{I}}\frac{\Delta \mathrm{I}}{\Delta\nu}=\frac{1}{3}\frac{\Delta \mathrm{I}}{\mathrm{I}}$$

becomes not only positive but large compared with v/c. The equation is then

$$\frac{1}{g}\frac{dv}{dt}=-\frac{2v}{c}+\frac{2}{3}\frac{\Delta \mathrm{I}}{\mathrm{I}} \quad . \quad . \quad . \quad . \quad (31)$$

The retardation becomes changed into an outward acceleration. It is precisely this outward acceleration which was the subject of my paper in *M.N.*, 1926 May. Of course the deduction here is subject to the condition that the atoms do not carry the absorption line with them.

* Apart from the wing-effect. It is shown below that this converts the retardation into an acceleration.

† The exact form of this is equation (14). Equation (30) follows by writing $\mathrm{I}\left(\nu_0\left(1+\frac{v}{c}\cos\theta\right),\theta\right)-\mathrm{I}+\frac{v}{c}\nu_0\cos\theta\frac{\Delta \mathrm{I}}{\Delta\nu}$ and proceeding as in the derivation of (17).

But if they do, as will be to some extent the case for prominences, the retardation will still be negligibly small.

Thus, whatever weight we attach to Pettit's interpretation of his observations as showing periods of unaccelerated motion, Sur's explanation of this breaks down. The v/c term as soon as it becomes important changes sign and gives rise to an acceleration, not a retardation.

9. *Average Doppler Effect for different Parts of the Sun's Disc.*—It may be observed that we have just shown that for a *fast* outward-moving atom, the equation of motion is approximately

$$\frac{1}{g}\frac{dv}{dt}=\frac{2}{3}\frac{v}{c}\left(\frac{\nu}{\bar{\mathrm{I}}}\right)_0\frac{\Delta \mathrm{I}}{\Delta\nu} \quad . \quad . \quad . \quad . \quad (32)$$

In my paper on accelerated emission, I adopted for simplicity a V-shaped line contour. With the notation there used, we have

$$\Delta \mathrm{I}=\mathrm{I}_1-\mathrm{I}_0,$$

and

$$\Delta\nu=\nu_0(\mathrm{V}/c),$$

where V is the Doppler velocity corresponding to the half-width of the line. Hence the equation of motion (32) is

$$\frac{dv}{dt}=\frac{2}{3}\frac{v}{\mathrm{V}}\frac{\mathrm{I}_1-\mathrm{I}_0}{\mathrm{I}_0}g \quad . \quad . \quad . \quad . \quad (33)$$

This differs from the equation there derived by the factor 2/3. It is easy to see the origin of this factor. The more detailed calculations of the present paper have taken account of the varying direction of the pencils of radiation incident on the atom, and the factor 2/3 arises from the varying Doppler effect with respect to radiation from different parts of the sun's disc. Light coming from the limb is perpendicular to the atom's motion, and the Doppler effect is zero. For $\alpha_\nu=3/5$ the factor has been seen to be 17/24; for $\alpha_\nu=1$, it is 3/4. As the atom moves further away from the sun's surface, the factor will, of course, approach unity. Accurate calculations for all distances from the sun could easily have been made. However, it seems sufficient to point out here that the estimates of the limiting velocity v_∞ made in the previous paper are slightly over-estimates.

10. *The observed Displacement of high-level Lines to the Red.*—St. John has shown * that in the sun the K_3 (central) component of the K line of Ca^+, at the centre of the sun's disc, is displaced to the red by an amount corresponding to a velocity of descent of 1·1 km./sec. This displacement decreases to zero from the centre to the limb, roughly according to a cosine law. Again, St. John and Adams,† from a study of high-dispersion spectrograms of Sirius, Procyon, and Arcturus, found that high-level lines in these stars showed a differential shift to the red (as compared with low-level lines) of an amount corresponding to a velocity of descent varying from 1·2 to 0·2 km./sec. for the three stars mentioned. Lastly, in his researches on the Einstein shift to the red,

* *Astrophys. Journ.*, **32**, 36, 1910.
† *Ibid.*, **60**, 43, 1924.

St. John has found * that, in addition to the relativity shift, there is at the centre of the disc a systematic shift which varies from +0·3 km./sec. for the most intense of the lines investigated to −0·2 km./sec. for the least intense; and he has shown that the effect is one which is to be correlated fundamentally not with line-intensity but with level. It appears that as the limb is approached, the systematic differences between high- and low-level lines disappear, but the wave-length shows an additional shift to the red which is known as the limb-effect.

St. John interpreted his observations on the K line as meaning a downward motion of the high-level Ca^+ vapour, of velocity 1·1 km./sec. Similarly, St. John and Adams interpreted the results on Sirius, Procyon, and Arcturus as meaning downward convection currents in the high-level atmospheres of these stars. Recently St. John has returned to the discussion, and concluded that "there can be little doubt that the interpretation based on the hypothesis of vertically descending currents is the correct one."

There are various difficulties about this interpretation. If the displacement to the red is a permanent feature, the atmosphere must be in a steady state. Whence come the atoms that replace those that fall? If the state were steady in the hydrodynamical sense, the hydrodynamical equation of continuity would require an ascending current to balance the descending one. But this would produce a Doppler effect in the opposite direction, which would neutralise the first one. For the ascending atoms would be just as capable of absorbing (and emitting) as the descending ones. In the older literature of astrophysics, gas is frequently spoken of as being in an "emitting" or "absorbing" state; but it must be remembered that each individual atom undergoes transitions from the emitting to the absorbing state and back again, under stellar conditions, at the rate of some 10,000 times a second.

St. John and Babcock† have discussed this difficulty. One explanation they suggest is that the currents are ascending over the small bright granules, descending over the larger dark interspaces, and that the observed effect is an integrated one. Another explanation is that the vapours which suffer an uprush in eruptive prominences diffuse horizontally through the action of high-level currents and gradually descend, in a manner "analogous to the volcanic action of Krakatoa and Katmai, and the spreading of the dust through the upper reaches of the terrestrial atmosphere." But a velocity of 1·1 km./sec. in the sun is not a "gradual descent." At this rate the whole chromosphere, 14,000 kms. thick, would settle in about 4 hours. The analogy with the dust of Krakatoa and Katmai seems to me a false one. Any interpretation in terms of convection currents has to explain how the enormous velocities of the order of 1 km./sec. are produced. I say "enormous" deliberately. Such velocities cannot by any stretch of imagination be described as a gradual descent. They correspond rather to a terrific vertical wind. At 5000°, the velocity of sound in calcium vapour is easily found to be 1·3 km./sec. Thus the supposed mass-motion is of the same order as the velocity of sound. No theory of diffusion could

* *Proc. Nat. Acad. Sci.*, **12**, 65, 1926.
† *Astrophys. Journ.*, **60**, 32, 1924.

account for such velocities. Nor could any theory of convection currents arising from unequal heating.

Both difficulties—the difficulty of maintaining the steady state and the difficulty of the size of the velocity—disappear if we replace the hypothesis of convection currents by the hypothesis of an asymmetrical velocity-distribution amongst the velocities of agitation of the individual atoms.

To get a line apparently shifted to the red, we must have a greater number of downward-moving atoms than of upward-moving ones. But the steady state can be maintained if the average downward velocity is less than the average upward velocity. Let $f(v)dv$ be the velocity-distribution. The equation of continuity requires that

$$\int_{-\infty}^{\infty} f(v)v\,dv=0 \quad . \quad . \quad . \quad . \quad (34)$$

In a symmetrical distribution we have $f(v)=f(-v)$. But it is possible for us to have $f(v)>f(-v)$ for v small and still have (31) satisfied; we merely have to impose in additon the condition $f(v)<f(-v)$ for v large. This is easily seen by writing (34) in the form

$$\int_{0}^{\infty} [f(v)-f(-v)]v\,dv=0.$$

The line produced by such an asymmetrical velocity-distribution would not, of course, be symmetrical. The minimum residual intensity (maximum absorption) will occur on the red side of the undisturbed frequency ν_0, but the intensity must increase a little more sharply on the red side than on the violet side. Whether this is in agreement with the facts remains for observation. It should be mentioned that St. John and Babcock's theory of ascending and descending currents whose effects are integrated also implies an asymmetry in the line and not a mere bodily shift of the line as occurs in the simple Doppler effect. Of course, the shifts in question are all very small compared with the widths of the absorption lines.

Another point in favour of an asymmetrical velocity-distribution is that the extent of the velocity-asymmetry might be expected to be of the same order of magnitude as the velocities of thermal agitation, in agreement with what is observed. This would agree with the observed increase of displacement with temperature from Arcturus to Procyon and Sirius.

Unfortunately, the investigation of the velocity-distribution amongst the high-level atoms given in this paper shows it to be a symmetrical Maxwellian one. I have scrutinised the analysis very carefully to see whether an asymmetrical distribution might possibly be brought into being. The equation of motion

$$\frac{1}{g}\frac{dv}{dt}=-\frac{2v}{c}\left[1-\tfrac{1}{3}\left(\frac{\nu}{I}\frac{\partial I}{\partial \nu}\right)_{\nu=\nu_0}\right] \quad . \quad . \quad . \quad (35)$$

might seem at first sight to necessitate an asymmetrical velocity-distribution. For it seems plausible to argue that each separate atom must

have zero acceleration, whatever its velocity, if we are to have a permanent chromosphere resting in equilibrium. This would require

$$1-\tfrac{1}{3}\left(\frac{\nu}{\mathrm{I}_\nu}\frac{d\mathrm{I}_\nu}{\partial\nu}\right)_{\nu=\nu_0}=0 \quad . \quad . \quad . \quad . \quad (36)$$

so that for $\mathrm{I}_\nu=r_\nu(2h\nu^3/c^2)(e^{h\nu/k\mathrm{T}}-1)^{-1}$ we should have

$$\left(\frac{\nu}{r_\nu}\frac{dr_\nu}{d\nu}\right)_0=\frac{h\nu_0}{k\mathrm{T}}>0.$$

The intensity-ratio r_ν would be increasing with ν at $\nu=\nu_0$, and the line would be displaced to the red, in agreement with observation.

This line of argument is tempting. But the analysis of § 7 shows that it is precisely because

$$1-\tfrac{1}{3}\left(\frac{\nu}{\mathrm{I}_\nu}\frac{\partial\mathrm{I}_\nu}{\partial\nu}\right)_{\nu=\nu_0}>0$$

that there is a steady state at all. Were (36) to hold, $\bar{v}^2$ would be infinite, by (27). This, however, only applies to the atoms at the highest levels. The exact contour of the absorption line is an integrated effect depending on the atoms at all levels. It is possible that the conditions lower down at the base of the chromosphere, where high-level equilibrium shades into equilibrium under an appreciable pressure gradient, require an asymmetrical velocity-distribution. But I am unable to adduce any arguments in its favour based on dynamics, in the absence of detailed calculations.

To get an asymmetrical velocity-distribution we want something asymmetrical amongst the various factors at work. The one asymmetrical effect which presents itself is the unilateral character of the emission of high-speed atoms under radiation-pressure. An outward-moving atom moving sufficiently fast to absorb in the violet wing of the line is driven off with an increasing acceleration; an inward-moving atom moving sufficiently fast to absorb in the red-ward wing of the absorption line is merely brought to rest. The clue to the explanation of the displacement to the red may lie here. The steady expulsion of outward-moving atoms, if it occurs, will lead to a deficiency in such atoms in the permanent chromosphere. Hence there will be fewer atoms contributing to absorption on the blue side than on the red, so giving an excess of absorption on the red side with a consequent displacement of the centre to the red. One would expect this effect to be too small to be appreciable; it is merely mentioned as an instance of how the one kind of asymmetry might lead to the other.

The observations of St. John and others definitely imply something asymmetrical about the motions of the upper layers of atoms, and whatever the nature of the asymmetry it has to be accounted for dynamically. It has seemed worth while to point out the possibility of an actual asymmetry in the distribution of atomic velocities as an alternative to convection currents, even though no dynamical evidence is brought forward in its favour, and though the investigation of the present paper indicates a symmetrical velocity-distribution for the atoms at the highest level.

11. *Summary* (1).—The force of radiation-pressure on an atom moving with a small velocity v is calculated. It is shown, following Einstein, that for values of v so small that the intensity-curve of the radiation in this Doppler range can be identified with its tangent, the atom is subject to an additional negative acceleration proportional to v. But for larger values of v, for which the contour of the absorption line in its wings becomes significant, the acceleration for an outward-moving atom becomes positive, and gives rise to the phenomenon of accelerated emission.

(2) For high-level atoms in a stellar atmosphere supported by radiation-pressure, the effect of the deceleration for small v, combined with the effect of randomness in direction of emissions and absorptions, is to set up a permanent velocity-distribution. The distribution of the radial components of velocity is Maxwellian. The state, however, is not one of thermodynamical equilibrium.

(3) If the effective temperature of the star is T_1, the Maxwellian velocity-distribution set up amongst the radial components of velocity is that corresponding to a temperature equal to $\frac{5}{8}T_1$, ignoring darkening towards the limb. When limb darkening is taken into account (T_1 now denoting the effective temperature at the centre of the disc) the factor $\frac{5}{8}$ is slightly reduced, but remains close to $\frac{1}{2}$.

(4) The application to prominence motions is considered. In agreement with R. K. Sur, it is shown that Pettit's conclusion as to the unimportance of radiation-pressure in causing prominences requires revision in the light of modern atomic theory. But Sur's suggestion that the pseudo-frictional resistance introduced by the motion may lead to uniform velocities is not confirmed.

(5) Some speculations are indulged in as to the possibility of the shift of high-level lines to the red, as observed by St. John and others, being due to an asymmetrical distribution of atomic velocities at lower levels. This hypothesis accounts for the facts, and removes some of the difficulties encountered by the hypothesis of convection currents.

Note added 1926 *July* 3.—The calculation given in the text refers only to the velocity-component radial to the star. Since the paper was communicated I have made a similar calculation for the cross-components of velocity. When the darkening towards the limb is taken into account the results are complicated, but the result for a uniform disc is simple and worth recording. It appears that in this case the mean square of the cross-component of velocity in a specific direction is given by $\frac{1}{2}m\bar{v}^2=\frac{1}{2}kT_1$, exactly as in thermodynamic equilibrium at a temperature T_1 equal to the effective temperature of the disc. On the other hand, the radial component has been seen to correspond to a temperature equal to $\frac{5}{8}T_1$. Thus the velocity-distribution is not spherically symmetrical. A more detailed calculation would be required to determine the velocity-distribution fully, but the results obtained indicate that the complete velocity-distribution is probably *ellipsoidal*, of the same character as that introduced by Schwarzschild in his theory of star-streaming. The ellipsoid has the form of an oblate spheroid with its short axis (axis of symmetry) radial to the star.

The Photometry of Astronomical Objects.

By A. F. and F. A. Lindemann. (Plates 11–13.)

General Principles.

The measurement of stellar radiation is important from two points of view, the statistical and the individual. The first obviously requires a knowledge of a large number of stars, for the essence of the statistical method is that the effect of deviations from the mean can be minimised by averaging over a large number of instances. It has been proved useful even with the ambiguous magnitude scale hitherto used, in drawing conclusions about the distribution of stars, their relative luminosities, and so on. The second point of view requires as exact a knowledge as possible of the fluctuations in brightness of individual stars; its value in helping us to understand many cosmological problems is exemplified by the researches on spectroscopic binaries, Cepheid variables, etc. From either point of view, the methods commonly employed appear capable of considerable improvement.

The ideal quantity to determine in either case would obviously be the spectral energy distribution-curve of each star. Unfortunately this solution is confined to a few of the brightest stars, since the light from faint stars, when spread out into a spectrum, can scarcely be accurately measured. Thus for statistical purposes one has not enough stars, and for individual purposes one is but seldom in a position to attain the necessary accuracy. Exact studies of the energy distribution-curves and their variations are, of course, invaluable; in view of its limitations, however, there seems room for another method applicable to a larger number of stars.

Of the other determinations at present in use, only one appears to have any real physical meaning, namely, the thermoelectric or bolometric measurement of the total intensity. The sensitivity of both these instruments is independent of the wave-length, all the incident energy is absorbed and converted into heat, and the rise in temperature, measured electrically, determines the total energy falling on the receiving surface. In the nature of things this process gives rise to a great deal of trouble. Small random fluctuations of temperature from the normal are almost impossible to exclude, and these being superposed on the rise in temperature, due to the incident radiation from the star, are bound to be a source of inaccuracy unless one is concerned with a very bright object. For this reason the measurement of the total radiation is confined practically to the bright stars whose spectral distribution-curve can be measured. If this latter quantity is available, it is of course to be preferred to the integral represented by a thermoelectric result.

All other methods of determining stellar magnitudes, *i.e.* the visual, photographic, or photoelectric, are strictly speaking devoid of physical significance. All that they give is the product of the solid angle subtended by the star, the emission of the star per unit surface of radiation of any wave-length, and the sensitivity of one's receiving instrument at this wave-length integrated over all possible wave-lengths from extreme

ultra-violet to extreme infra-red. (It is assumed, of course, that the effects of absorption and scattering in the atmosphere and instrument are eliminated, though this is by no means invariably the case.) It is evident that any magnitude determinations are meaningless unless the sensitivity-curve of the sensitive surface, be this the visual purple, the photographic plate, or a hydrated alkali metal, is defined, and even when this is done such a determination is highly ambiguous. Only if one is comparing stars of equal temperature or spectral class can these magnitudes be reasonably interpreted, and it is significant that in modern publications the spectral class is now usually stated when a star's magnitude is mentioned.

The detects of the above-mentioned instruments are in a sense a source of strength. Precisely because they are sensitive only in the neighbourhood of the visible region of the spectrum, they are immune to random fluctuations of temperature or low frequency radiation. For this reason they are susceptible of great accuracy, even if very faint stars are to be measured ; their use, however, can only be justified subject to certain restrictions.

Were it possible by their means to measure the brightness or variation in brightness of one or more wave-lengths in a star's radiation, this would obviously be extremely valuable, as it would be a step on the way to the ideal solution, namely, the determination of the energy-curve. Since this would entail spreading the light out into a spectrum with all the attendant loss of energy, it is scarcely feasible. As a compromise, it is suggested to employ more or less monochromatic filters. These can be obtained, and reproduced, almost transparent to light in practically any desired band of wave-lengths, whilst practically opaque to all the rest. If such filters were agreed on and standardised, magnitudes determined through them would have some physical significance. Unless the transmission band were very broad, the sensitivity-curve of the receiving surface would be immaterial, since only comparative measurements are involved, and the luminosity of stars could be stated in ergs per cm.^{2} within certain wave-lengths by standardising one's instrument with the help of a black body at a known temperature. In the following, a description is given of a photoelectric photometer designed for this purpose.

Apparatus.

In the *Monthly Notices of the Royal Astronomical Society* 1919,* a description was given of a preliminary apparatus designed and made at Sidmouth for the photoelectric measurement of the magnitudes of stars and other telescopic objects. Though some parts of the new apparatus are unchanged it has, in many respects, been materially improved. It is hoped that a description of the new features may prove useful to others working, or contemplating working, in this or allied fields.

Obviously, any astronomical photometry presupposes a telescope. The photometer which has been developed simply takes the place of the eyepiece. It consists essentially of the cell-box containing the photoelectric cell or cells, and the electrometer, by means of which the electric

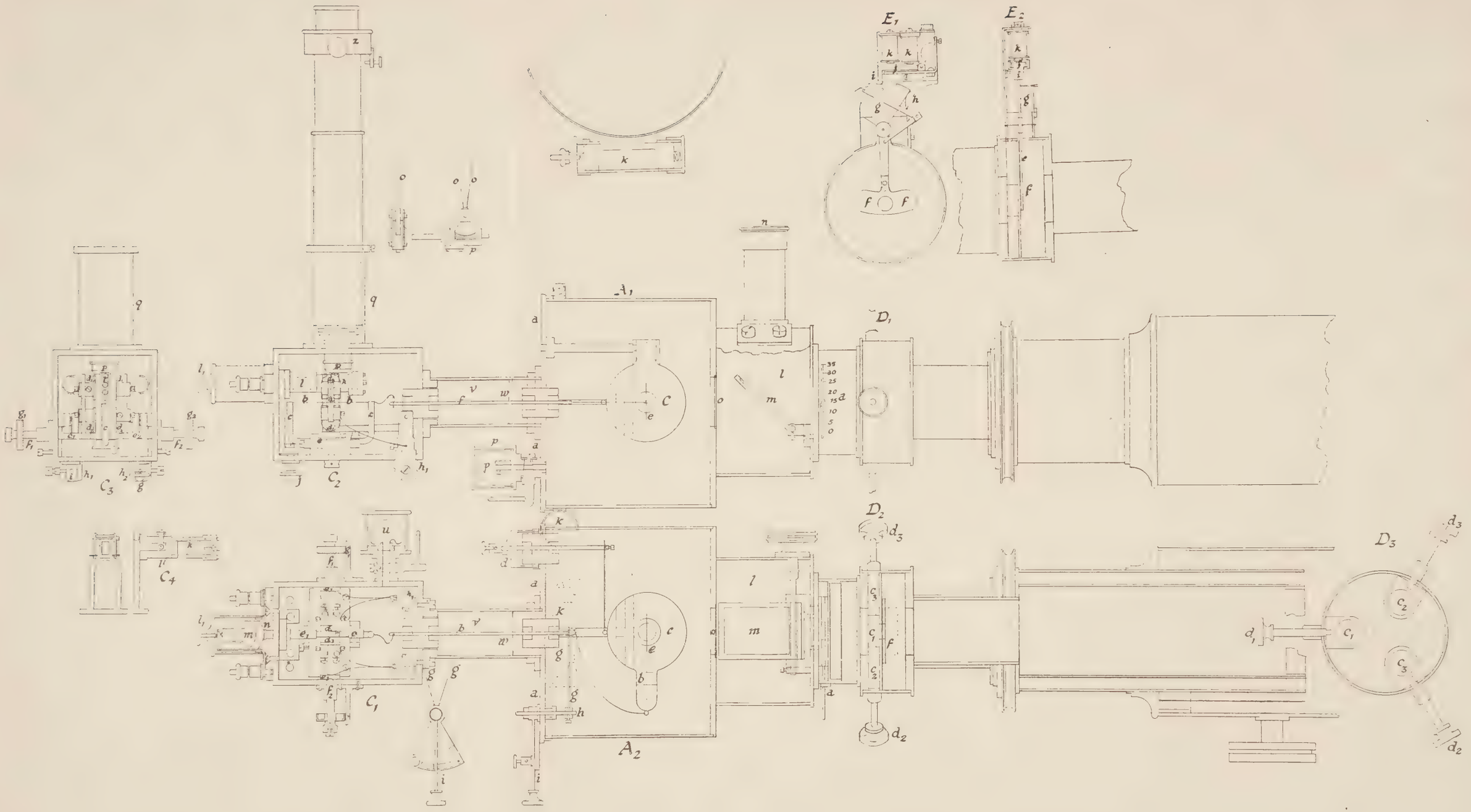
* *M.N.* **79**, 343.

current, produced by the cell under the influence of light, is measured. Since the current is accurately proportional to the incident light, for any given wave-length, the strength of the current is an accurate measure of the amount of light. In addition to these two main parts there are various subsidiary parts necessary, or at any rate useful, for accurate work. The first and most essential, in view of what has been stated above, is an arrangement to allow the interposition of various filters, usually more or less monochromatic, in the beam of light. The second is an electrically worked shutter which allows the light to fall on the cell for an accurately measured time. Further there are, of course, an ordinary eyepiece, which can be thrown in when required to make sure one has the right object in the field of view, a diaphragm in the focal plane to cut out extraneous light, a standard cell to make sure the sensitivity of the electrometer is constant (though this proved almost superfluous with the new type of electrometer), and a photographic attachment for recording accurately the current measured by the electrometer.

The Cell-box.

Two cell-boxes are shown in the drawing (Plate 11), the smaller, A, being the same as was used previously and described in the paper cited above. It consists essentially of the base-plate (*a*) carrying the photo-electric cell (*c*) by the earthed ring (*b*) which grips the guard-ring on the cell. As will be remembered, the cell has an internal, as well as an external, guard-ring. In the older cells this was a silvered ring on the inside of the tubular part of the cell. In the later cells this has been replaced by a ring of platinum wire sealed on to the inside of the tube. The potassium of the cell is connected by a lead insulated with amber and covered with ebonite to keep out light where it passes through the base-plate to the terminal (*d*), which is joined to an external source of high negative potential (some 200 volts). The ring or grid (*e*) is connected to the electrometer by means of a thin wire attached to the lead (*f*) to the electrometer. Two wires (*g*) at an angle of some 30° form a V capable of rotating for an angle of some 60° at the point of junction about an axis (*h*) perpendicular to the plane of the V and of the base-plate. Their motion is governed by a small external lever (*i*) with an insulated silica end. At the one extreme position the external lever makes contact with an earthed metal plate, and the one arm of the V presses upon the lead (*f*) to the electrometer, so that electrometer and grid are earthed. At the other extreme position the external lever makes contact with a plate connected with the negative pole of the battery giving the standard potential, so that the electrometer reads this voltage. In the intermediate position the external lever is earthed, but neither of the arms of the V touches the lead to the electrometer, which thus assumes the voltage of the grid.

The base-plate is covered by a brass cylinder which carries the intermediate standard battery (*k*). A 1·4 volt battery from a pocket lamp was found to be remarkably constant, and was quite satisfactory provided it was compared periodically with a standard voltage. A smaller cylinder (*l*), in which a plane mirror (*m*) rotating through 45°


E1
E2
A1
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C1
C2
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C4
D1
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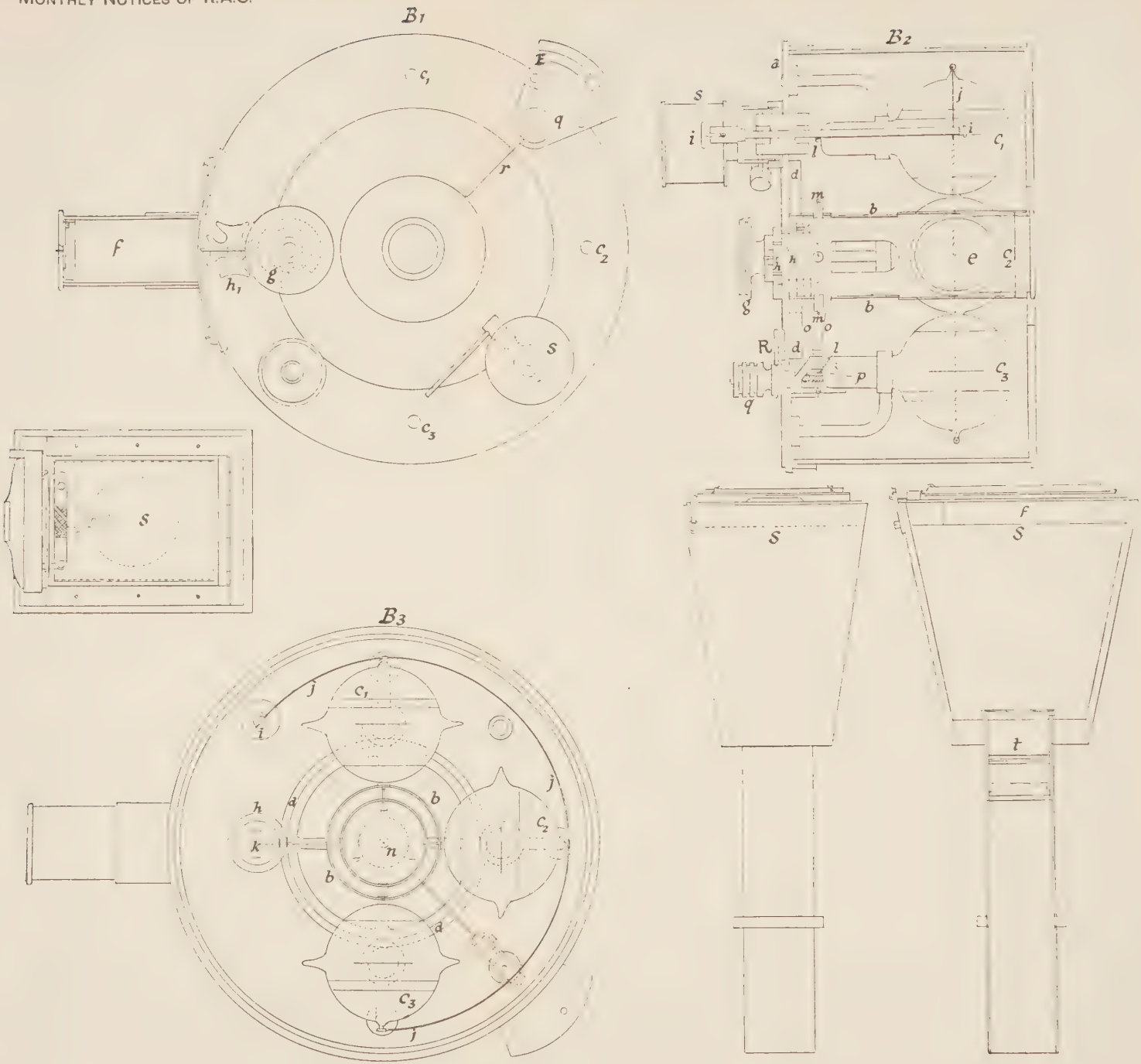
B1
B2
B3
c1
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E
q
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allows the image of the star to be thrown into the eyepiece (*n*) when required, is soldered to the other end of this cell-box. A thin glass plate (*o*) is fixed between the two compartments to prevent interchange of air with the air in the cell-box, which is kept dry by the drying-box (*p*) containing interchangeable glass vessels (*q*) filled with metallic sodium. The far end of this compartment holds the iris diaphragm and leads to the filter-box and shutter, which will be described later.

The new cell-box, B_1, B_2, B_3 (Plate 12), is designed to allow the use of three different cells, which one sometimes finds it convenient to employ when the ordinary potassium cell is not sufficiently sensitive to the light emitted by the star in the region of the spectrum to be studied. In this box also the cells, C_1, C_2, C_3, are mounted on a base-plate (*a*) held by their guard-rings in a vertical position at an equal distance from, and facing, the centre of the cell-box. In the centre of the cell-box is a short brass tube (*b*) carried on a toothed wheel (*d*) which can revolve about the axis of the cell-box. This tube carries a plane mirror (*e*) at 45° to the axis, arranged so that the light from the star which enters through a hole at the top of the cell-box is reflected at right angles to the axis. By rotating the mirror the light can be thrown as desired on to any one of the cells, or into the eyepiece (*f*), B_1. This last, fixed to the side of the cell-box, enables one to observe the star or other object, and adjust its position in the field. The tube and mirror are rotated from outside the cell-box by turning the knurled head (*g*), which is fixed to a small cog-wheel (*h*) with one quarter of the number of teeth of the main wheel (*d*). One complete turn of this head (*g*) therefore moves the image from the eyepiece to one of the cells, or from one cell to the next. A special gear (*h'*) and a pointer enables one to set the mirror so that the light is thrown into exactly the same spot each time. The electrical connections are all carried by the base-plate. The external negative potential is brought in through the insulated wire (*i*), which is permanently joined to the cathodes of all the three cells by the wire (*j*). The connection to the anodes or grids of the cells is made by a small plate (*k*), which presses elastically against the anode (*l*) of the cell into which light is being reflected by the mirror. This plate is carried by a radial wire attached to a ring of copper wire (*m*), which is attached by silica supports to the brass tube (*b*) and revolves with it. This ring is connected to the wire leading to the electrometer by means of a spring contact (*n*). The cell can be earthed or connected to the standard voltage by means of a V-shaped wire (*o*), lying in a radial plane of the cell-box. A worm drive (*p*), connected to an external knurled head (*q*) carrying a metal lever (*r*), enables it to be turned about the point of the V. In one extreme position one arm of the V presses against the ring (*m*), while the lever (*r*) makes contact with the negative pole of the standard battery; in the other extreme position the other arm of the V makes contact with the ring (*m*) while the lever (*r*) is earthed. In the intermediate position, of course, the ring is free, and the cell and electrometer are connected together but insulated from the box. As in the other cell-box the intermediate standard battery is fixed to the cell-box, and the drying-box (*s*) to the base-plate (*a*). The top of the box screws straight on to the carrier of the iris diaphragm, to which, as before, are fixed the filter-

box and the shutter. It is, of course, possible to fix filters permanently in front of the various cells, in which case the filter-box can be dispensed with, but in general it is more convenient to be at liberty to use the filters or not as desired.

The Electrometer.

It was pointed out in the previous paper on photoelectric photometry (*M.N.*, 1919 March), that a certain amount of difficulty was experienced because the electrometer's zero and sensitivity depended upon the position of the instrument. In other observatories this difficulty has been surmounted by hanging the electrometer in a vertical position on to the telescope by means of a Cardani suspension. This way out has many disadvantages besides its inherent clumsiness, weight, and so on. The cell must of necessity be fixed relative to the telescope, so that if the electrometer is to move there is some difficulty in keeping the capacity constant. The measure of the incident light is, of course, the quantity of electricity set free, and this is measured by the change of potential of the electrometer fibre. It is clear, therefore, that this change of potential is only proportional to the incident light if the capacity is constant. Even more important from the practical point of view is the fact that it is almost impossible to maintain the highest insulation unless all insulators are kept carefully dry. When cell-box and electrometer are rigidly fixed together this is readily achieved, but if they are to be capable of relative motion it is almost impossible.

For these reasons it seemed worth while to design an electrometer whose zero and sensitivity should be independent of position. The design C_1, C_2, C_3 decided upon has proved so sensitive, accurate, and convenient that it has now been adapted for general physical use (see *Phil. Mag.*, 1924 March). The general principle is that of the quadrant electrometer, the main innovations being the use of a very light fibre (*a*), some 2 cm. long and 20μ thick, as needle, which is fixed to the middle of a silica torsion fibre (*b*), about 4 cm. long and 4μ thick, kept under a tension equal to about half its breaking strain on a frame (*c*) of the same material. Since this needle, which incidentally is made of two fibres sealed together at the ends, one on either side of the torsion fibre, can be accurately balanced, the restoring force is simply the torsional elasticity of the fibre, and quite independent of gravity, so that the position of the electrometer is immaterial. In the instrument shown the quadrants are replaced by plates (d_1, d_2, d_3, d_4) suitably cross-connected and mounted on silica rods fitted to the slides e' e''. These can be made to approach one another, or to separate, by means of the screws (f_1, f_2). The screws carry divided heads and scales (g_1, g_2) which enable one to reproduce any desired setting. Motion of the quadrants or plates is only required to vary the field. Although this is extremely convenient it can, of course, be dispensed with, and the field varied by changing the applied voltage, for instance by means of a potentiometer. The field is applied to the plates through the insulated terminals (h_1, h_2), which also serve as feet if one wishes to disconnect the electrometer and use it for some other purpose. The earthing terminal (*i*) forms a third foot,

and the fourth foot (*j*) contains a screw in case the electrometer has to stand on an uneven surface. In this electrometer the motion of the cross fibre or needle is observed through a small 45° prism (*k*) supported on a hollow ebonite carrier (*l*) (see C_4). The light which can be switched on and off at (*l*), from a small 3-volt lamp (*m*), talls through a red glass (*n*) through a hole in this carrier on to the end of the fibre. The light can be run from a dry cell of the type used in pocket batteries, but in general it is more convenient to connect it to an accumulator by a piece of ordinary flex. Immediately above the prism (*k*) is the objective (*p*) of a microscope whose tube fits into the tube (*q*) permanently fixed to the electrometer-box lid. With this microscope one can either use an eyepiece with a scale (*r*) or, if a permanent record is required, a small camera (*s*) taking $4\frac{1}{2} \times 6$ cm. plates. In this camera one fixes a scale and lens at (*t*), and takes two superposed photos by switching on the lamp (*m*) before and after exposing the cell to the light of the star. By this means an accuracy of about 1/5 per cent. is readily obtainable. If desired, of course, a third photo of the fibre can be taken with the standard voltage. The whole electrometer box is of brass and air-tight, and is dried by metallic sodium in the drying-box (*u*).

The lead from the electrometer to the cell-box is carried by amber insulators from the centre of the tube (*v*). Ebonite plates between rings (*w*) prevent any leakage of light from the electrometer to the cell. The capacity of the whole system, cell, connections, electrometer needle is about 3 to 4 cm. with the old cell-box A, and slightly greater with B.

Diaphragm, Filter-box, Shutter.

Since it is clearly necessary to have but one object sending light into the cell at any time, it is necessary, whichever cell-box one uses, to place a diaphragm in the focal plane of the objective. It may be convenient to take this, the filter-box, and the electric shutter together (D). As diaphragm an ordinary iris diaphragm was used, calibrated before assembling with an external scale (*a*), which enabled the aperture to be adjusted from outside. The diaphragm was opened wide in general while observing by eye, and closed down until all extraneous light was excluded, before attempting a photometric determination.

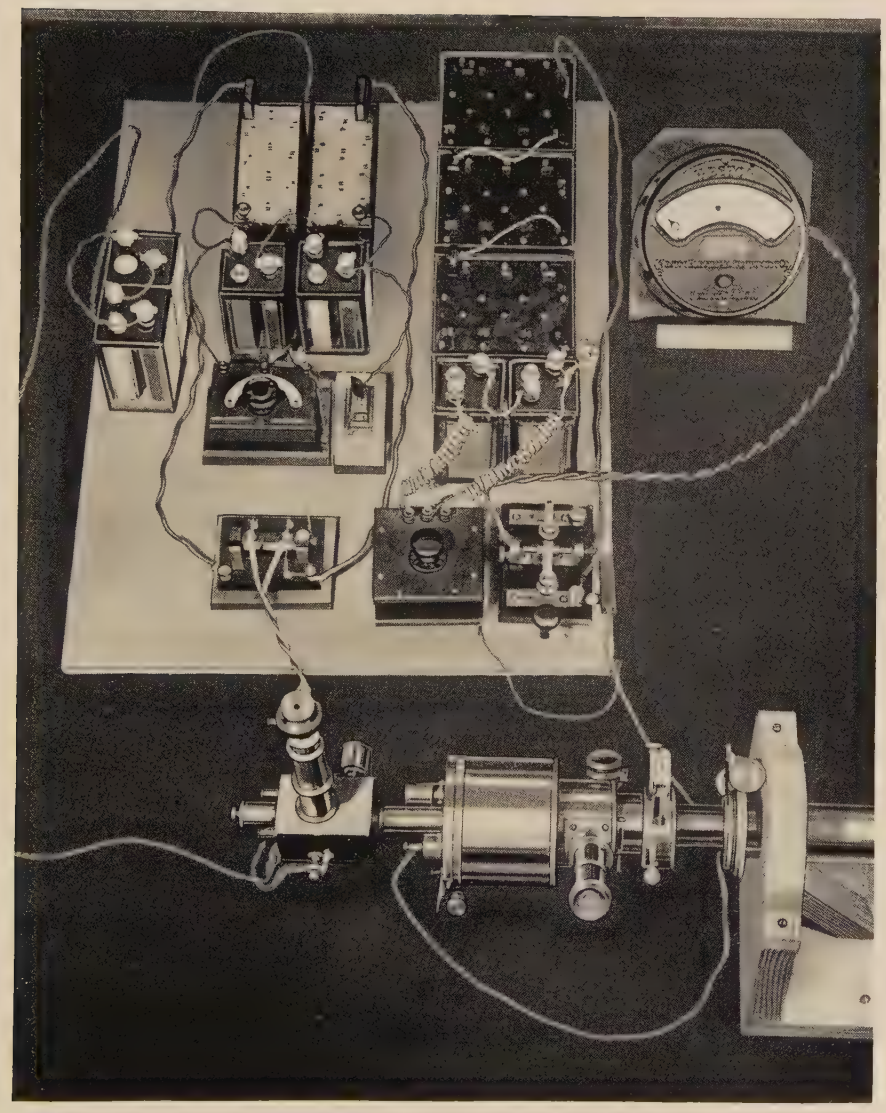
The filter-box (*b*) contains three filters (c_1, c_2, c_3). The filters are carried on rods which slide in tubes and end in knobs (d_1, d_2, d_3) painted the colour of the filters to avoid mistakes. When the rods are all pulled out there is a free passage for the light through the middle. By pushing any of the knobs in to the full extent, the filter is interposed in the path of the beam. The filters selected in the first place are transparent in a fairly narrow band, one in the red, one in the green, and one in the violet part of the spectrum. The width of these bands is such that with a potassium cell and sunlight one obtains approximately the same current from each.

This is convenient, as one can see at once whether a star is hotter or colder than the sun by the ratio of the deflections. An accurate determination of the temperature can, of course, be made, as pointed out in the above-mentioned paper, by reference to the calculated curve.

Since very accurate readings could be made of the potential, and since the capacity being absolutely constant, this is an accurate measure of the incident light, it became necessary to make sure that the time during which the cell was exposed to the light could be regulated accurately. For this purpose an electric connection with a clock having a seconds pendulum was an obvious suggestion; the only novelty perhaps consisted of the introduction of an aluminium wheel, with 60 equidistant holes into which metal pegs could be stuck, which made contact with a platinum spring at intervals of any desired whole number of seconds. This contact was in series with a contact made by a second pendulum, so that an accuracy of a small fraction of a second could be attained. The current was not sufficiently strong to excite an electromagnet which could release an ordinary camera shutter. It was undesirable to introduce a relay or other complicated amplifying device, and finally a new shutter (E_1, E_2) was made. This consists of a light, balanced, aluminium plate (*f*), the inner end having a hole of appropriate size in the centre. The outer end consists of a sector-shaped piece (*g*) with three teeth or notches. This is held over to one side by the spring (*h*). For use it is pulled over to the other side, and held there by the catch (*i*) which engages the third tooth or notch. The catch (*i*) is fixed to the anchor (*j*) of the electromagnet (*k*). When the magnet is activated by the current through the contact on the clock, the shutter snaps over to the central position where the catch falls into the middle notch, and the hole being in the axis the light can pass on to the cell. When the next contact comes round, the catch is released again, and the beam of light is interrupted.

Accessories.

Since the previous paper various improvements in accessories, *i.e.* sources of potential, etc. have been made. The technical manufacture of high-tension batteries for wireless telephony has brought about a great simplification in photoelectric work. Similarly, potentiometers, high-tension resistance voltmeters, are now cheap and good. Plate 13 shows the complete apparatus with all necessary batteries, potentiometers, and so on. There are three separate circuits—the lighting circuit, the electrometer circuit, and the cell circuit. The first consists simply of two accumulators, in series, connected to the electrometer lamp, which has its own switch. The second consists of two 30-volt batteries, joined by two accumulators, which discharge through a plug-in switch and a potentiometer. The movable point of this is earthed, so that when the wires are plugged into any desired potentials on the batteries, the zero can be moved as convenient. This is really superfluous, as the same can be attained by adjusting the quadrants as described, but it is often less trouble, as it cannot affect the sensitivity. The terminals connected to the electrometer plates can be connected with or insulated from these batteries, or such fraction of them as appears desirable, through the mercury two-way switch. The third circuit consists of four accumulators, in series, discharging through a potentiometer whose movable point is earthed. To their negative pole is connected the positive pole of a system of five sixty-volt wireless batteries, in series, into



which an insulated cable, leading to the metal in the photoelectric cell through the terminal on the cell-box, can be plugged. By moving the potentiometer one can obtain a fine adjustment of this applied voltage through a range of eight volts. The exact voltage used, which must be kept constant to ensure comparable results, is shown by the voltmeter with a resistance of 250,000 ohms, which can be thrown in by a switch when required.

With this apparatus very accurate measures can be made of the emission of radiation from stars or other astronomical objects within any desired region of the spectrum. Which bands of wave-lengths should be chosen, and what filters should be standardised, is, of course, a matter for international discussion and agreement. That the method is feasible is certain, that some reconsideration of our photometric system is necessary we believe we have shown. It is to be hoped that the radical change involved will not prevent its consideration and adoption.

The Spectrum of the Bright-hydrogen-line Star H.D.C. 20336 in Camelopardalis (Sp. Type B3pe). By Major William J. S. Lockyer, M.A., Ph.D. (Plate 14.)

In a previous communication to the Society * on " Recent Observations of some Bright-hydrogen-line Stars " a brief account was given of the behaviour of the bright hydrogen lines as observed in certain B-type stars taken between 1923 September and 1924 February. It was there pointed out that the star H.D.C. 20336 in Camelopardalis (R.A. $3^h11^m.2$. Dec.+65° 17′. Mag. 4·76. Sp. Type B3pe) was evidently periodic in nature, this statement being based on the recent changes recorded in relation to earlier published observations.

Since the last date mentioned other photographs of the spectrum of this star have been secured during the routine programme of photographing the spectra of these bright-hydrogen-line stars with the 9-inch prismatic camera of the Norman Lockyer Observatory which has a dispersion of 46·0 mm. between H_β and H_δ. These photographs show such striking changes in the hydrogen lines that an approximate period for this star can be deduced.

Historical.

The discovery of the bright-line nature of this spectrum was made on 1895 September 22 by Espin,† who recorded " F bright ? ".

Mrs. Fleming ‡ in her publication of " Stars having Peculiar Spectra," which has no date, gives the credit of the discovery to Espin.

In a presumably earlier publication § (no date is given), under the

* *M.N., R.A.S.*, **84**, 413, 1924.
† *Astr. Nachr.*, **140**, No. 3352, 243.
‡ *Ann. Harvard Coll. Obs.*, **56**, No. 6, 182.
§ *Ibid.*, **56**, No. 2, 33.

title " Stars having Spectra of Class B," Mrs. Fleming includes this star in her catalogue, with a note to the effect that " The hydrogen line H_β is bright in this star."

Campbell* published in the *Astrophysical Journal* for October 1895 a paper on " Stars whose Spectra contain both Bright and Dark Lines," and under the heading of this star he recorded that he observed H_α as bright, but gives no reference to any previous observation.

A good early detailed description of the spectrum of this star is given by Miss Maury † from negatives taken in the years 1885 and 1886 in a volume entitled " Spectra of Bright Stars." This account is of great importance in the present investigation, so the note may be quoted in full :—

" The system of dark lines in the spectrum of this star belongs to Group IV., division 6, but all the lines are exceedingly wide. A distinct bright band lies almost centrally on the wide dark H_β. A fainter and narrower bright band or line lies upon H_γ, its position being commonly of slightly greater wave-length than the centre. The lines H_δ and H_ϵ appear in most photographs unequally divided, with the strong component occupying the true central position for each hydrogen line, and the weak and hazy component having the greater wave-length. In some plates, however, both these lines and H_γ appear triple. Other lines of the spectrum sometimes appear unequally divided, with the stronger component of shorter wave-length."

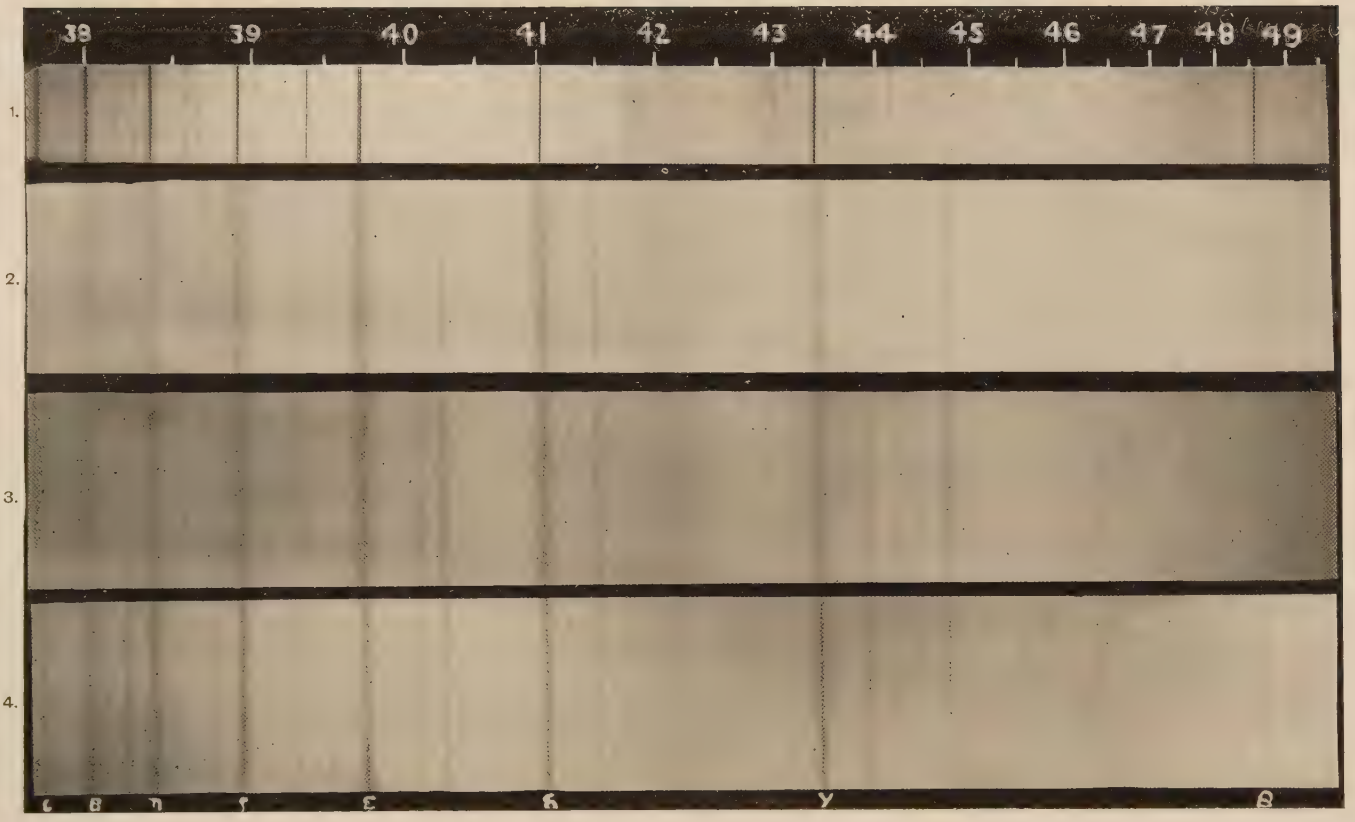
Dr. Merrill,‡ in his dissertation on " Class B stars whose spectra contain bright hydrogen lines," photographed the spectrum of this star in 1912 July and August and found " H_α strong bright," " H_β and H_γ double bright on broad absorption," " Strong helium absorption," and " Traces of a few bright ϕ Persei lines." Unfortunately Merrill did not give any idea of the relative intensities of the bright components of these hydrogen lines.

Dr. Curtiss of the Detroit Observatory seems to have made the most detailed investigation of the spectrum of this star so far, which he describes under the title " Changes in the spectrum of H.R. 985." §

The negatives he discussed can be grouped into two epochs, namely, 1912 October to December and 1916 January to April. He publishes in full the measures of the brightness of parts of the emission components of H_β, H_γ, H_δ, H_ϵ, and H_ζ referred to the adjacent continuous spectrum, and in another table he gives the measured velocities, line intensities, and line widths of the same lines.

He shows that marked changes were observed between the years 1912 and 1916 in the relative intensities of the bright hydrogen components, but could not detect any certain change within each series of plates in either year. Again, within each series of plates, no change in the measured velocities could be established, but a relative displacement of forty-five kilometres was well justified between the observations of 1912 and those of 1916.

* *Astroph. Journ.*, **2**, 180, 1895.
† *Ann. Harvard Coll. Obs.*, **28**, Part 1, 105, 1897.
‡ *Lick. Obs. Bull.*, No. 237, 166, 1913.
§ *Pub. Astr. Obs. Univ. Michigan*, **2**, 39, 1916.



1. α Cygni. 2. and 3. H.D.C. 20336, 1924 Oct., 1925 Oct. 4. γ Orionis.

He concludes by stating that at least two bodies are involved, forming a system with a long orbital period, possibly about 2·3 years.

The only other remaining observations of this star known to the writer are those published in a communication to the *Astronomische Nachrichten* * by Dr. A. Hnatek on "Radial Velocity Observations with the Vienna Coudé spectroscope."

Dr. Hnatek included H.D.C. 20336 among the stars he observed at Vienna, and he gives the values he obtained on two occasions in one year. Thus on 1918 November 23 the velocity he measured was +38·6 km. while on December 16 it had increased to +46·2 km.

Dr. Hnatek inclines to consider this change of velocity as real and not due to error of measurement, for he states in a note :—

"Da die einzelnen Linien untereinander gut stimmen und einzelne ganz schwache Linien die Differenz beider Geschwindigkeitsbestimmungen sogar noch zu vergrössern scheinen, dürfte der Unterschied reell, die Geschwindigkeit also variabel sein."

The Sidmouth Observations, 1923–1925.

The Sidmouth photographs were taken between 1923 September 10 and 1925 December 17. They divide themselves into four groups, namely : 1923 September 10 to 16, 1923 December 12 to 1924 January 27, 1924 October 23 to November 17, and lastly 1925 October 14 to December 17. In each group the spectra are practically all similar, but between them there is a progressive difference.

The appearances and changes in the hydrogen lines are very similar to those recorded in the writer's two earlier papers on ϕ Persei † (see Plate 14).

Each hydrogen line consists of a very broad absorption band in the centre of which is a bright emission band about a third of the width of the absorption band. This emission band is split up centrally by a sharp absorption line, making the emission appear as two bright lines symmetrically situated on the broad absorption band. So far as can be judged from the negatives those bright lines are always symmetrical as regards the centre of the broad absorption band. This observation is of importance when reference is made to Miss Maury's description. The distance between the bright lines or components of each hydrogen line increases as the violet is approached, just as the widths of the broad absorption bands become greater.

The dispersion of the Sidmouth instrument is such that the bright components of the hydrogen lines are well separated, so that eye estimates or measures by the wedge or both can be readily made. It may be remarked that none of the photographs taken are as good as those secured for ϕ Persei for the reason that the star's magnitude is 4·76 and therefore 0·57 magnitudes fainter than ϕ Persei, while its type is *B*3 and consequently less actinic than the Bo type of ϕ Persei (see Plate 14).

The accompanying table (Table I.) gives in columns (1) and (2) the

* **210**, No. 5030, 246, 1919.

† *M.N., R.A.S.*, **85**, 580, 1925 ; **86**, 474, 1926.

Norman Lockyer Observatory numbers of the negatives with the dates on which they were secured. Column (3) states the condition exhibited by the bright hydrogen lines as regards motion in the line of sight described later, (N) denoting negative velocity and (P) positive velocity. In columns (4) and (5) are stated the estimated relative intensities of the bright red (R) and violet (V) components for the two lines H_β and H_γ, the symbol (>) meaning "brighter than," and (>>) "very much brighter than."

TABLE I.

N.L.O. Neg. No.	Date.	Vely. Km.	Visual Estimates. H_β.	Visual Estimates. H_γ.	Wedge Measures H_β. R.	Wedge Measures H_β. V.	Diff. R–V.	Means. R–V.	Ratio $\frac{R}{V}$.	Authority of Photographs.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
	1885 Dec. 2 1886 Jan. 7	N	..	R>V*	..	..	..	..	..	Harvard.
	1886 Nov. 26 27 Dec. 2 16	N	..	R>V*	..	..	..	..	..	,,
	1912 Oct.–Dec.	− 3·1	R>V	R>V	1·5†	1·2†	..	..	1·25	Curtiss.
	1916 Jan.–Apr.	+41·7	V>R	V>R	1·0†	1·5†	..	..	0·66	,,
	1918 Nov. 23	+38·6	V>R	V>R	..	..	..	..	..	Hnatek.
	Dec. 16	+46·2	V>R	V>R	..	..	..	..	..	,,
357	1923 Sept. 10	N	R>>V	R>V	13·1	11·5	+1·6			
363	11	N	R>>V	R>V	14·4	12·4	+2·0	+2·0	1·16	Lockyer.
369	16	N	R>>V	R>V	15·1	12·7	+2·4			
408	Dec. 12	N	R>V	R>V	13·5	13·0	+0·5			
411	19	N	R>V	R>V?	..	..	..	+0·5	1·03	,,
413	20	N	R>V	R>V	12·3	11·3?	+1·0?			
426	1924 Jan. 27	N	R>V	R>V	13·6	13·7?	−0·1?			
498	Oct. 23	P	V>R	..	13·0‡	13·7‡	−0·7	−0·7	0·95	,,
516	Nov. 17	P	V>R	..						
690	1925 Oct. 14	P	V>>R	V>>R	11·9	15·1	−3·2	−3·2	0·79	,,
779	Dec. 17	P	V>>R	V>>R	..	..	..	..	..	,,

In columns (6) and (7) are given the wedge measures of the intensities of the components of these pairs for H_β when they were measurable, and columns (8) and (9) the intensity differences from those measures and their means respectively. In order to compare these intensity differences with the earlier observations of Curtiss, the figures in columns (6) and (7) have been converted into ratios, and the mean values for each

* Deduced from Miss Maury's description of the Harvard negatives.
† Intensities of bright components referred to continuous spectrum (Curtiss).
‡ Wedge estimates, not measures.

group of observations, which are bracketed together, are given in column (10).

The four values of these intensity ratios are plotted in fig. 1, with these ratios as ordinates and a scale of years as the abscissæ.

It may be pointed out here, as was shown in my two previous communications * on the spectrum of ϕ Persei, that the curve representing the intensity differences between the red and violet components of the hydrogen emission lines could be converted into one representing the velocities in the line of sight. Above the zero line, or what is the same thing in this case, the unit ratio line, the conditions represented are those in which the red (R) component is brighter than the violet (V) component, or a state of negative velocity: below this line the conditions are the reverse, namely, the violet component is stronger than the

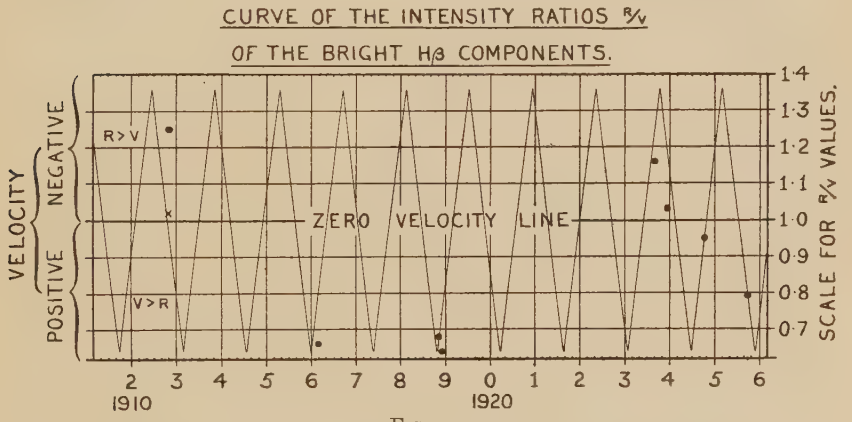


Fig. 1.

red, and the velocity in the line of sight is positive. The above statement will be found very useful in locating the positions of some of the observations on the curves to which reference is made later in this paper.

Reverting to the Sidmouth observations, it was found that in drawing curves to satisfy these values, one outstanding feature was that a curve representing the variation of 2·3 years, suggested by Curtiss, could not be made to fit the facts.

An attempt was therefore made to embody, as far as possible, all the previously recorded observations. Thus Hnatek's observations extended the time interval back five years from the first Sidmouth observations in 1923. Curtiss's data carried it back eight and eleven years respectively, while Miss Maury's reaches the comparatively long time interval of thirty-seven and thirty-eight years before 1923.

Reference will now be made to each of the observations in turn, dealing first with those made by Hnatek.

* *M.N., R.A.S.*, **85**, 584, 1924, and **86**, 474, 1926.

Hnatek's Observations of 1918.

Dr. Hnatek deals only with the spectroscopic velocities in the line of sight, but these values give important information for the determination of the star's period. Both his observations indicate that the velocities recorded by him had a large positive value, namely +38·6 km. for 1918 November 23 and +46·2 km. for December 16 of the same year. This latter value is approximating to the order of magnitude of greatest positive velocity, as is judged by the maximum positive velocity of ϕ Persei, namely +45·5 km.*

Any curve satisfying the Sidmouth observations must therefore indicate a maximum of positive velocity at the approximate date towards the end of the year 1918. The velocities published by Hnatek are inserted in Table I. column (3) with the dates of observation in column (2).

Curtiss's Observations of 1912 *and* 1916.

Curtiss not only gives the direction and amount of the velocity for the two epochs 1912 October to December and 1916 January to April, but he also adds the relation between the intensities of the components of the bright hydrogen lines with regard to the adjacent continuous spectrum. From the latter, then, it becomes possible to state approximately the ratios of the intensities of the components for each hydrogen line.

In Table I., columns (4) and (5), are shown the estimated relative intensities of the bright components for H_β and H_γ for his two epochs, and in columns (6) and (7) the measured intensities of the bright components of H_β in relation to the adjacent continuous spectrum. The ratios of these intensities are added in column (10).

Curtiss's observations show, therefore, that his 1912 group indicates negative velocities (R>V), and must therefore be represented as falling on the velocity curve above the unit-ratio or zero-velocity line, and his 1916 group (V>R) must fall on the curve below this line.

Further, for the latter group (1916) the point on the curve must be approximating to a minimum owing to the large positive velocity measured by him, namely +41·7 km., again assuming that this velocity is near the value of maximum positive velocity, as in the case of ϕ Persei mentioned previously.

Again, while the intensity ratio for his 1912 group is comparatively large, the recorded measured velocity is small, *i.e.* only −3·1 km.

It is difficult to reconcile these values, because the record of such a small velocity would indicate that the red component of the hydrogen emission line would be only slightly brighter than the violet component, while he gives quite a large value deduced with respect to the neighbouring continuous spectrum. The fact that the velocity he records is so consistently low in all the plates Dr. Curtiss measured seems to suggest that it may be looked upon as having greater weight than the intensity value.

* *M.N., R.A.S.*, **86**, 485, 1926.

Bearing this in mind, both these observations of intensity-ratio are plotted in fig. 1 at their respective dates, but a cross has been also added at the probable position as regards intensity ratio of the 1912 observation.

Miss Maury's Observations, 1885 and 1886.

Coming now to Miss Maury's description of the Harvard photographs which fall into two groups, namely those taken at the ends of each of the years 1885 and 1886 (see Table I.), it must first be noted that the scale of the spectra under investigation was very small. Thus while the dispersion of the 11-inch Draper telescope, with the one prism that was used, was only 2 cm. between H_β and H_ϵ, that of the Sidmouth prismatic camera is 5·7 cm. for the same region, or 2·85 times greater.

It was not therefore possible for Miss Maury to see the detailed structure of the hydrogen lines in the negatives she examined. Thus in the case of H_β she would see the broad absorption, in about the middle of which she would observe a bright line, but she could not detect that the bright line was really a bright band with a sharp absorption line at its centre.

Nevertheless, Miss Maury has so accurately described what she observed that it is not difficult to translate her descriptions in the light of the present knowledge of the appearances and changes in progress in the structure of the hydrogen lines, especially as her description of the spectrum of ϕ Persei, written at the same time, can be compared with the now known spectrum changes in this star.

In order to test exactly what can be observed in a small-scale spectrum of a bright-hydrogen-line star, the writer reduced two photographs of the spectrum of ϕ Persei (dispersion 5·7 cm. H_β to H_ϵ) to exactly the same scale (2·0 cm. H_β to H_ϵ) as that used by Miss Maury. Photographs of H.D.C. 20336 were not used as they do not attain the quality of those of ϕ Persei. They exhibit, however, the same structure and changes in the hydrogen lines, so that Miss Maury's descriptions will equally apply.

The negatives selected were taken at the epochs of maximum negative and maximum positive velocities, which may be called (A) and (B) respectively, or, in other words, they represent typical cases where in (A) the red emission components were very strong relative to the violet components, and where in (B) the violet components were very strong relative to the red components.

The negatives were photographed alongside each other, both being of the same scale, and they were so adjusted that the broad hydrogen absorption bands were made coincident. It should be remarked that this reduced spectrum is probably a better spectrum than any of those examined by Miss Maury, because it is reduced from one of very good definition in the laboratory, while those used by her were taken directly at the telescope and atmospheric tremors would probably mar, to some extent, the definition in spectra on so small a scale.

We may now state what is actually observed in these small-scale spectra in comparison with Miss Maury's descriptions of the H_β and H_γ lines.

Taking H_β first. This line appears in the reduced spectrum as a narrow absorption band, in the centre of which is a distinct bright line —the actual duplication can just be detected. It would be difficult with certainty to say from either negative whether the bright line was displaced from its central position in the absorption band, but there seems a tendency for it to be slightly displaced towards the red in photograph (A) and towards the violet in (B).

Miss Maury's observation, therefore, that "a distinct bright line lies almost centrally on the wide dark H_β" would apply equally to both photographs.

Coming now to the line H_γ, in which it should be remembered that the bright components are more widely separated, here again the bright components in (A) and (B) are visible (separated by a relatively weak absorption line), although the outstanding feature in each is the great prominence of the brighter component in each case.

In photograph (A) this bright line, relative to the broad absorption, appears distinctly shifted towards a position of greater wave-length, but in (B) it appears equally shifted towards the position of smaller wave-length. Further this bright line is weaker and narrower than that observed at H_β because the latter is a blend of two lines.

Miss Maury's description of H_γ is as follows :—

"A fainter and narrower bright band or line lies upon H_γ, its position being commonly of slightly greater wave-length than the centre." She further states that in some photographs H_γ appears triple.

This latter remark shows that on some photographs both bright components were seen making the broad absorption band appear as three absorption bands or lines. The quotation given above indicates that she did not observe the condition of the line as exhibited in photograph (B) but that shown in (A). In (A) the velocity in the line of sight is negative or the red component is brighter than the violet component (see Table I.).

From this it may with little doubt be concluded that Miss Maury's statement that the bright band or line on H_γ was "*commonly* of slightly greater wave-length than the centre" suggests that there were some negatives which showed this bright band as being near the central position. Thus some negatives may have been taken near maximum negative velocity and others nearer the zero-velocity line. It is, however, quite impossible to say which negatives belong to the former or to the latter case, but it may be fairly concluded that none of them indicated positive velocity.

Miss Maury's description of H_γ is quite in keeping with the observed appearance in the reduced spectrum of this hydrogen line in ϕ Persei. Her description of the H_δ and H_ϵ lines, which "in some plates appear triple," shows that the greater separation of the bright components, as the violet is approached, rendered it possible for her to observe these bands of absorption, while she did not recognise that this triple appearance was due to the presence of two bright lines on the broad absorption band.

Determination of the Star's Period.

A periodic curve to satisfy all the above observations must therefore comply with the following conditions :—

(1) The Harvard observations (1885 and 1886) must represent negative velocities and be included in that portion of the curve lying above the zero-velocity line.

(2) Curtiss's groups (1912 and 1916) must represent negative and positive velocities and must fall above and below the zero-velocity line respectively. Also the 1916 group must be near a minimum of the curve, but the 1912 group may probably fall just a little above this line.

(3) Hnatek's observations of 1918 distinctly show that they must fall close to a minimum of the velocity curve or be near or at maximum positive velocity.

(4) The Sidmouth observations (1923 to 1925) must represent a progressive change from negative to positive velocity (not necessarily all in one cycle), with the latest value not far from a minimum of the curve.

Numerous attempts were made to try curves of various periods to satisfy all the above conditions, but the only one that was fairly satisfactory was that having a period of one year and five months or 1·4 years, with epochs of maximum and minimum specified on the time scale at the bottom of fig. 1.

Such a curve is indicated in fig. 1, but it does not include the Harvard observations as the diagram would have to be reduced to too small a scale.

While it is obvious that all the observations of this star are too few in number to form an accurate normal curve, yet it was considered advisable to draw an approximate one omitting only the deductions made from Miss Maury's statements.

For this purpose the zero phase selected was that of 1924 July 1, and the following table (Table II.) shows the computed phases for each of the observations.

TABLE II.

Date.	Photographs.	Intensity Ratios R/V.	Phase. Months.
1885 Dec. 20	Maury	R>V	13·7
1886 ,, 3	,,	R>V	8·0
1912 Nov. 15	Curtiss	1·25	13·5
1916 Mar. 1	,,	0·66	2·0
1918 Dec. 4	Hnatek	V>>R	1·0
1923 Sept. 12	Lockyer	1·16	7·4
,, Dec. 27	,,	1·03	10·9
1924 Nov. 4	,,	0·95	4·0
1925 Oct. 14	,,	0·79	15·5

As regards the ordinates of the points plotted, these have been placed where the intensity ratios were known, as given in the above table,

except in the case of Curtiss's 1912 observation, which has been indicated just above the zero velocity line to show the small negative velocity recorded by him (see fig. 2). In the other case Hnatek's observation, owing to the large positive velocity he measured, is placed at the intensity-ratio value 0·62. Miss Maury's points were not considered when drawing the curve, but were placed near the curve at their appropriate dates. The letters against the plotted points indicate the first letters of the observers' names.

The curve shows a maximum about midway between the minima and indicates a kink in the downward side to make it conform more to the observations at those phases. It fulfils very well all the conditions enumerated in the beginning of this section and so indicates that the period of 1·4 years seems to satisfy the observations very well.

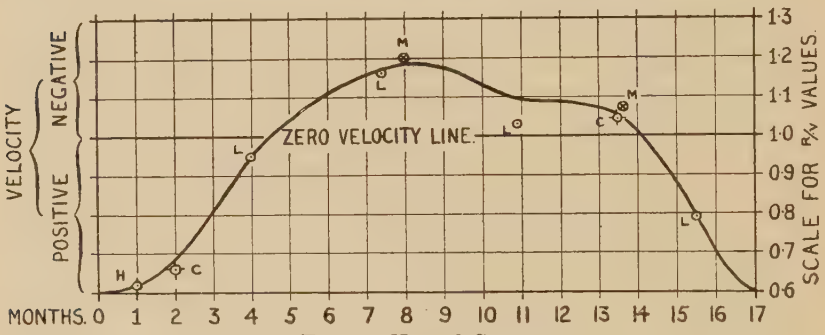


FIG. 2.—Normal Curve.

Other Lines in the Spectrum.

Helium.—In addition to the broad series of hydrogen lines with their intricate structure, the spectrum shows another series of broad absorption lines due to helium. These lines belong to the diffuse and sharp doublet systems and the diffuse singlet system, a table containing the wave-lengths of which was published in a previous account of the spectrum of ϕ Persei in which they were present * (see Plate 14).

These lines, however, behave quite differently from those recorded in ϕ Persei. Thus, for instance, the lines of the diffuse doublet system, namely λ4471·6, λ4026·3, and probably λ3819·7, all show broad bands of absorption with a single bright line centrally placed on them, and this appearance is clearly shown in the best negatives. In no photograph do these lines vary their appearance as regards sharpness and diffuseness as they do in the case of ϕ Persei. The peculiar behaviour of the line at λ4471·6, as recorded in a previous paper on the spectrum of ϕ Persei,† namely, the splitting up of the sharp absorption line into two well separated absorption lines, is not seen in the present photographs.

* *M.N., R.A.S.*, **85**, 596, 1925.
† *M.N., R.A.S.*, **86**, 493, 1926.

Three lines of the sharp doublet systems, namely λλ 4713·1, 4120·8, and 3867·5, are also seen in the spectra, but in this case as absorption lines only.

The lines of the diffuse singlet system, namely λλ 4387·9, 4143·8, and occasionally λ 4009·3, all behave in exactly the same manner as the lines of the diffuse doublet system.

Miss Maury evidently noted something peculiar about those lines, as she stated that "other lines of the spectrum appear unequally divided." Merrill seems to have missed the bright lines, as he only refers to these lines as "strong helium absorption." Perhaps they may be variable. Curtiss refers to them as "sometimes giving evidence of the structure of the characteristic of the hydrogen lines," and enumerates the wave-lengths of some of those lines found in the Sidmouth photographs.

Magnesium.—The ionised magnesium line at λ 4481·3 appears in some of the Sidmouth negatives as sharp and moderately strong. In the spectra of φ Persei it was almost on the verge of invisibility, excepting at the epoch of greatest hydrogen absorption, when it became a little stronger * (see Plate 14).

No previous observer seems to have made any mention of the presence of this line in the spectrum.

Calcium.—The presence of the K line, or the absorption line of ionised calcium at λ 3933·8, is noted in some of the photographs as of a sharp nature. Its intensity is generally moderate to faint, but like the other lines may possibly be variable. In one negative it was seen as being probably double bright or having bright borders, as was often the case in the spectrum of φ Persei (see Plate 14).

Curtiss records that "a narrow dark K line, probably fixed in position, was observed on many plates."

Bright Ionised Lines.—The bright ionised lines of iron and titanium, so prominent a feature of the spectrum of φ Persei, are here only just visible, as the writer remarked in a previous publication † from an examination of negatives taken in 1923 September and December and 1924 January. The same remark applies to the Sidmouth photographs taken since then. These lines were evidently invisible on the plates examined by Miss Maury, but Merrill recorded traces of them, and Curtiss not only says that they showed faintly the structure characteristic of the hydrogen lines but measured four of them.

It is more than probable that when a greater number of photographs of the spectrum of this star have been taken systematically throughout a complete period, cyclical changes both in the intensity and character of the above-mentioned lines would become evident which are now only hinted at.

Further, the fact that very similar types of variations are taking place in the spectrum of this star and φ Persei, in spite of the great difference in the lengths of their periods, leads one to the conclusion that whatever may be the causes which produce the one must be the basis of the origin of the variations of the other.

* *M.N., R.A.S.*, **86**, 494, 1926.
† *M.N., R.A.S.*, **85**, 592, 1925.

Summary.

1. This paper contains a discussion of the spectra of the bright-hydrogen-line star H.D.C. 20336 (Sp. Type B3pe) taken during the years 1923 to 1925 at the Norman Lockyer Observatory.

2. A brief summary is given of previous observations relating to the spectrum of this star.

3. Conclusions drawn from all the observations suggest a period of variation of 1·4 years, but fail to corroborate the tentative period of 2·3 years put forward by Curtiss in 1916.

4. The presence of other lines in the spectrum is discussed, and it is pointed out that the appearance and behaviour of the helium lines is unlike those recorded in the spectrum of ϕ Persei.

Description of Plate.

Plate 14. A comparison is here given between two spectra Nos. (2) and (3) of H.D.C. 20336 and (1) α Cygni (typical of ionised metallic lines), and (4) γ Orionis (typical of helium lines). They are enlarged ($\times 2$) reproductions, broadened by the clepsydra apparatus, of negatives numbered 498 and 690, details of which are given in Table I.

The Use of a Standard Equinox in Astronomy.

By L. J. Comrie, M.A., Ph.D.

Abstract.—Instead of changing our equinox of reference continually, the use of the equinox of 1950·0 is advocated as a standard for cometary and asteroid work. The general specifications of the data which should be tabulated by the *Nautical Almanac* in order to facilitate the adoption of such a standard equinox are outlined, and later the details are developed, with special reference to the modern tendency towards the use of calculating machines. A new solution of the corrections for differential precession, nutation, and aberration is offered, also special facilities for reducing right ascension, declination, longitude, latitude, or orbital elements to 1950 or vice versa. Methods of computing special perturbations are discussed on the basis that the planetary material for the best method should be tabulated. The advantages of Cowell's method for general use, with Encke's as an auxiliary, are emphasised.

Introduction.—It is the practice to refer the solar, lunar, planetary, and stellar ephemerides to the "apparent equinox of date." For certain observing purposes this is essential. For other purposes a moving plane of reference has disadvantages—for instance when we wish to compare two meridian observations of the same star, or to define the orbital plane of a comet by reference to the plane of the ecliptic. Hence arises the idea of the ".mean equinox of the beginning of the year." Even this restriction of the number of possible reference planes is often insufficient, and we find the star observations of a number of years reduced to a common equinox—for modern observations generally that

of 1900·0 or 1925·0. It is the worker in dynamical astronomy who is most perturbed by changing equinoxes, and until very recently the national ephemerides have given him little help in the way of providing data referred to a standard equinox. In the following paragraphs the idea of a standard equinox is advanced, with details of the "accessories" which it is suggested should be given by the *Nautical Almanac* to facilitate the use of this equinox.

In choosing a standard equinox it is, of course, highly desirable that the necessity for changing it should not arise for some considerable time. This points to a date in the future, and that of 1950·0 is suggested. This will serve till about 1980, when it can be changed to 2000·0.

The idea of a standard equinox is not new. Ristenpart * in 1906 proposed 1875 as the "Normal Equinox" of the nineteenth century and 1925 for the twentieth. This suggestion was the result of his experience with the *Geschichte des Fixsternhimmels*, and related to star catalogues. His proposal has now been superseded by the I.A.U. recommendation that the equinox of reference for catalogues containing meridian circle observations should be changed once every twenty-five years. The *Berliner Jahrbuch* supplies material enabling the equinox of 1925 to be used in minor planet work, while the *Connaissance des Temps* provides for the equinox of 1930. But neither of these ephemerides gives in full information that meets the general requirements outlined below.

Banachiewicz † has given coefficients for reducing rectangular co-ordinates from any equinox to that of 1925, and Bower ‡ has prepared similar tables for reduction to 1900. These are based on formulæ developed by Oppolzer. As their use involves nine multiplications and three additions for each transformation it is evident that the publication of rectangular co-ordinates referred to a standard equinox will save the computer considerable labour. Bower prefers the equinox of 1900, for the very good reason that it is the equinox of the astrographic catalogue and charts, as well as of many modern catalogues. But as astronomers will probably wish a change of equinox once every fifty years it is too late now to adopt 1900.

Modern Tendencies.—Before proceeding to detail the information which should be tabulated to make the adoption of a standard equinox feasible, particularly for researches on cometary and asteroid orbits, we advert to the necessity of catering for two modern tendencies in computing. The first of these is the use of calculating machines instead of logarithms. To be sure the mechanical calculator is not by any means established in astronomy (although it is used more in Germany and in the United States than in this country) but it is making headway, and is bound to come more and more into favour. However, until the ephemerides tabulate the natural values of many quantities that are now given as logarithms, the full advantages of a machine cannot be realised. *Hence the ephemerides must lead, and machines follow.*

The other tendency, which goes hand in hand with the one just

* *V.J.S.*, **41**, 232, 271.
† *Rocznik Astronomiczny Obserwatorjum Krakowskiego.*
‡ *A.J.* In press.

mentioned, is the decimal division of the degree. The advantages of some decimal division are obvious. The centesimal division of the quadrant, a French product, has never thrived, and is now dying a lingering death, with what is probably its death-blow dealt by a Frenchman himself—by Andoyer in his wonderful single-handed production, the *Nouvelles Tables Trigonométriques Fondamentales*. On the other hand, the decimal division of the nonagesimal degree has been revived and is flourishing. First introduced by Briggs in the *Trigonometria Britannica*, it received an immediate setback by Vlacq's *Trigonometria Artificialis* with its 10″ interval. Only within the last half-century has it been re-established by the efforts of Bremiker, Lohse, and Peters, who have provided tables rendering its use possible. It is particularly in the problems of celestial mechanics that it can be used to advantage.

General Requirements.—It is desirable that means shall be provided to enable astronomers—

(1) To compute cometary and asteroid orbits referred to the ecliptic or equator of 1950.
(2) To give ephemerides of comets and minor planets referred to the equator and equinox of 1950.
(3) To reduce differential observations to 1950 for comparison with ephemerides, or for the determination of orbits.
(4) To reduce observations referred to the mean equinox of any year to the standard equinox of 1950, and vice versa.
(5) To reduce longitude, latitude, or orbital elements from any annual equinox to that of 1950, and vice versa.
(6) To compute perturbations throughout long periods without the necessity for applying precessional and nutational corrections to the planetary longitudes and latitudes now given.
(7) To refer the motion of a comet or asteroid, if desired, to the centre of gravity of the solar system.

Data for the Sun.—The material proposed to afford these facilities will now be discussed. The longitude, latitude, and radius vector data are illustrated by a sample page heading with two lines entered. This shows at a glance the number of decimals to be given, the differences, signs, etc. The figures are, of course, fictitious, although of the right order.

Sun 19xy. Mean Equinox of 1950·0.

Date	Longitude		Latitude		Radius Vector	
Jan. 1	279°·12345	7′ 24″·4	+314	11″·3	1·0123456	
	1°·12345	4044″·4				+1234 −44
2	280 ·24690	14 48 ·8	317	11 ·4	·0124690	

The degrees of longitude are not repeated in the column giving the minutes and seconds. The first column under latitude is in units of 0°·00001. Although the latitude referred to the ecliptic of date seldom exceeds 1″, it may attain approximately $\pm 0''\cdot 5\ (1950-t)$ when referred to the ecliptic of 1950. The quantity following the first difference of the radius vector is the mean second difference. The natural radius

vector is given here as log radius vector is already provided for elsewhere in the almanac. The unit 0°·00001 corresponds to 0″·036, but as the Sun's longitude and latitude are computed from Newcomb's Tables to 0″·01 this unit can safely be adopted. It corresponds to 7-figure accuracy in trigonometrical work.

In the case of the Sun's co-ordinates X, Y, Z (which would be tabulated for every midnight only), first and second differences would be given throughout with their signs. The formation and use of third differences will be avoided if the computer interpolates with a modified Everett's formula, utilising the two second differences corresponding to the dates between which he is interpolating. The coefficients required would be given in a separate table to four decimals for every 0·01 of the interval. This completes the material necessary for requirements (1) and (2), and which would fill sixteen pages annually.

Differential Precession, Nutation, and Aberration.—When the position of a comet or other moving body is compared with that of a star by means of a micrometer, the observer measures the differences of R.A. and Dec. of the two bodies referred to the apparent equinox of date. Call these $\Delta\alpha$ (in minutes of time) and $\Delta\delta$ (in minutes of arc), taken in the sense comet minus star. To obtain the position of the comet referred to the equinox of 1950 we add these differences to the mean position of the star for 1950, together with corrections for proper motion, if known, and for differential precession, nutation, aberration, and refraction. The latter correction is outside the scope of this paper. While the other corrections are frequently neglected when the position of the comet is referred to the mean equinox of the beginning of the year, it is not possible to neglect differential precession when referring to a more distant equinox.

Formulæ and methods for computing these differential corrections have been given by Ristenpart,* Becker,† and Merton,‡ while tables have been given by Ristenpart,§ Merton,‡ and in the *Berliner Jahrbuch*. None of these appear to be entirely satisfactory. The tables of the *B. J.*, first introduced in 1927, are for reduction to the beginning of the year of observation only, and are of triple entry. As even double entry tables are not popular, many observers will doubtless not care to use three tables of triple entry for these small corrections. For this reason the following modifications are offered.

The differential aberration is separated from the other two corrections for three reasons : (1) because it is constant each year and hence lends itself to a permanent table ; (2) because means will be given to enable the observer to reduce for differential precession and nutation either to the beginning of the year of observation or to 1950, and (3) because this correction is so small that it will usually be neglected. Its maximum value for two stars separated by 1° is of the order $0^s{\cdot}02 \sec\delta$ in R.A., or 0″·3 in declination.

* *A.N.*, 3832–33.
† Valentiner's *Handwörterbuch der Astronomie*, IIIa, 239.
‡ *M.N.R.A.S.*, **85**, 513.
§ *Loc. cit.* Also in later numbers of the *A.N.* and in *Observatorio Astronomico de Santiago de Chile, Publicationes No.* 1.

Taking the Besselian formulæ for aberration, *i.e.*

$$\begin{array}{ll} \text{In R.A.} & Cc + Dd \\ \text{In Dec.} & Cc' + Dd' \end{array}$$

differentiating, reversing signs to give the reduction from apparent place to mean place, and writing

$$C' = C \sin 1^{\mathrm{m}} \quad D' = D \sin 1^{\mathrm{m}}$$

we get for the corrections due to differential aberration

$$\text{In R.A.} = (C' \sin\alpha - D' \cos\alpha) \frac{\sec\delta}{15} \Delta\alpha - (C' \cos\alpha + D' \sin\alpha) \frac{\sec\delta \tan\delta}{225} \Delta\delta$$

$$\text{In Dec.} = (C' \cos\alpha + D' \sin\alpha) \sin\delta \, \Delta\alpha + (C' \sin\alpha - D' \cos\alpha) \frac{\cos\delta}{15} \Delta\delta$$

In the second equation a small term of $-0''{\cdot}0024 \cos\odot \sin\delta \, \Delta\delta$ has been neglected as it would never amount to $0''{\cdot}1$ in any observation. Putting

$$C' \sin\alpha - D' \cos\alpha = F(\alpha)$$

we see that

$$C' \cos\alpha + D' \sin\alpha = F(\alpha + 6^{\mathrm{h}})$$

so that the formulæ become

$$\text{Diff. aberration in R.A.} = F(\alpha) \frac{\sec\delta}{15} \Delta\alpha - F(\alpha + 6^{\mathrm{h}}) \frac{\sec\delta \tan\delta}{225} \Delta\delta$$

$$\text{,,} \qquad \text{,,} \qquad \text{,, Dec.} = F(\alpha + 6^{\mathrm{h}}) \sin\delta \, \Delta\alpha + F(\alpha) \frac{\cos\delta}{15} \Delta\delta$$

The function $F(\alpha)$ can be tabulated on two pages (facing one another) in a permanent double entry table for every fifth day, and for every $0^{\mathrm{h}}{\cdot}5$ of α up to 12^{h}, after which the function repeats itself with reversed sign. Adopting as the unit 0·01, the maximum value of the quantity tabulated is ± 9, so that no interpolation will be needed in using the table. In ordinary observations where $\Delta\delta$ does not exceed 10′, and particularly of asteroids where δ does not exceed about 40°, the only term which need be applied is the first term of the correction in R.A., and then only when $\Delta\alpha$ is large. It must be borne in mind that the position of a comparison star is rarely known to 1″, still more rarely to $0''{\cdot}5$, while errors of observation are of the same order or greater in the case of comets without a well defined nucleus, so that the refinement of applying corrections of the order of $0''{\cdot}1$ or even $0''{\cdot}2$ is scarcely warranted.

For the differential precession and nutation we turn to the formulæ involving the independent day numbers f, g, and G. After differentiating and putting

$$g \sin 1^{\mathrm{m}} = j$$
$$G - 90^{\circ} = J$$

we get for the differential correction required

$$\text{In R.A.} = j \sin(J + \alpha) \frac{\tan\delta}{15} \Delta\alpha - j \cos(J + \alpha) \frac{\sec^2\delta}{225} \Delta\delta$$

$$\text{In Dec.} = j \cos(J + \alpha) \Delta\alpha$$

If A and B are the Besselian day numbers on any date in the year t, then

$$j \sin J = 20{\cdot}043(1950 - t - A) \sin 1^{m}$$
$$j \cos J = B \sin 1^{m}$$

from which j and J are readily found. j is of the order $\frac{1950 - t}{12}$, and changes fairly regularly and slowly as does also J, which is near 6^{h} before 1950 and near 18^{h} after 1950. It will be sufficient to tabulate j to 0·01 and J to 1^{m} for every tenth day of the year. If these occupy three columns of a left-hand page (with the formulæ at the foot) the balance of the page could contain a table of

$\frac{\tan}{15}$	$\frac{\sec^2}{225}$	$\frac{\sec}{15}$	$\frac{\tan \sec}{225}$	sin	$\frac{\cos}{15}$
3	4	2	3	2	2

to the number of decimals shown under each function. The right-hand page facing could contain 3-figure sines and cosines for every 1^{m}. Thus two consecutive openings would give all the formulæ and material necessary for these small corrections, and in a convenient form.

It should be remarked that the value of α to be used in the formulæ above is strictly the R.A. of the star for $\frac{1950 + t}{2}$ increased by $\frac{\Delta\alpha}{2}$. To avoid the necessity for reducing α to the half-way epoch it will be sufficient to add $1^{s}{\cdot}5(t - 1950)$ to the value of J computed as above before tabulation. This amounts to using the term m of the annual precession in R.A. and neglecting the term $n \sin\alpha \tan\delta$. In the corresponding corrections to δ it will usually be sufficient to apply simply $\frac{\Delta\delta}{2}$, and then only when δ or $\Delta\delta$ is large.

Reduction from Annual Equinoxes to 1950·0.—The formulæ

$$\alpha = \alpha_0 + (\bar{m} + \bar{n} \sin\bar{\alpha} \tan\bar{\delta})(1950 - t)$$
$$\delta = \delta_0 + \bar{n} \cos\bar{\alpha}(1950 - t)$$

where α and δ are for equinox 1950, α_0 and δ_0 for equinox t, and $\bar{m}$, $\bar{n}$, $\bar{\alpha}$, $\bar{\delta}$ for equinox $\frac{1950 + t}{2}$, have the disadvantage of requiring a fairly exact knowledge of $\bar{\alpha}$ and $\bar{\delta}$ at the mid-epoch. Hill's formulæ involving ζ_0, z, and θ are only required when the interval of reduction is long, or when the star is near a pole. The formulæ proposed by Ristenpart * have the following advantages :—

(1) They are more rigorous than the m and n formulæ.
(2) They involve only functions of the right ascension and declination at the initial epoch, and not at the mid-epoch.
(3) They lend themselves to easy tabulation of the coefficients required.
(4) The tables given for reduction *to* 1950 will also serve for reduction *from* 1950 (unless the interval is very great).

* *Observatorio Astronomico de Santiago de Chile, Publicationes No.* 1.

These formulæ are quoted in full here as they are far less well known than they deserve to be.

$$\alpha = \alpha_0 + A + A_1 \tan\delta_0 + A_2 \tan^2\delta_0$$
$$\delta = \delta_0 + D + D_1 \tan\delta_0$$

where

$$A = \zeta_0 + z + \frac{\sin^2\theta \sin 2\alpha}{4 \sin 1''}$$

$$A_1 = \frac{\sin\theta \sin\alpha}{\sin 1''}$$

$$A_2 = \frac{\sin^2\theta \sin 2\alpha}{2 \sin 1''}$$

$$D = \frac{\sin\theta \cos\alpha}{\sin 1''}$$

$$D_1 = -\frac{\sin^2\theta \sin^2\alpha}{2 \sin 1''}$$

in which $\alpha = \alpha_0 + \zeta_0$. Annual 3-page tables giving A_1 and D for every 1^m of α_0 up to 12^h (after which repetition occurs), and A, A_2 and D_1 for every 10^m suffice for accurate and convenient reduction to 1950 when precessions and secular variations are not available.

For reduction from 1950, using accents to denote quantities found from these tables with argument $24^h - \alpha$, we have

$$\alpha_0 = \alpha - A' + A_1' \tan\delta - A_2' \tan^2\delta$$
$$\delta_0 = \delta - D' + D_1' \tan\delta$$

An observer who wished to reduce an ephemeris for the equinox of 1950 to the equinox of date for the purpose of setting an equatorial would simply apply the terms A' and D' mentally.

In addition to the annual tables commencing (say) in 1930 three extra tables for reduction from 1875 (equinox of the G.F.H.), 1900 (equinox of the Astrographic and many modern catalogues), and 1925 (equinox of the most recent catalogues) should be provided.

Reduction of Longitude, Latitude or Orbital Elements to 1950.—In reducing longitude λ and latitude β or orbital elements Ω, i, and ω from one equinox to another by the usual methods (*e.g.* Bauschinger, *Tafeln zur theoretischen Astronomie*, Table XXX.) the approximate values of these quantities for the mid-epoch must first be determined, likewise those of the precessional constants involved, viz. p (annual precession in longitude), Π (node of moving on fixed ecliptic), and π (rate of change of inclination of moving to fixed ecliptic). Also a multiplication by the number of years in the interval of reduction is called for.

These inconveniences may be avoided if we tabulate on a single page values of a, b, c, d, where

$$a = \bar{p}(1950 - t), \quad b = \bar{\pi}(1950 - t), \quad c = 180^\circ - \Pi + \frac{a}{2}, \quad d = \frac{b^2}{2} \sin 1''$$

The formulæ of application are found to be

$$\begin{aligned}
\lambda &= \lambda_0 + a - b\cos(\lambda_0 + c)\tan\beta_0 - d\sin 2(\lambda_0 + c)\tan^2\beta_0 \\
\beta &= \beta_0 + b\sin(\lambda_0 + c) - d\cos^2(\lambda_0 + c)\tan\beta_0 \\
\Omega &= \Omega_0 + a - b\sin(\Omega_0 + c)\cot i_0 + d\sin 2(\Omega_0 + c)\cot^2 i_0 \\
i &= i_0 + b\cos(\Omega_0 + c) + d\sin^2(\Omega_0 + c)\cot i_0 \\
\omega &= \omega_0 - (\text{sum of 3rd and 4th terms in } \Omega)\sec i_0
\end{aligned}$$

In actual practice, since d amounts to only 0″·005 in a century, the terms in d will be small unless β_0 is large or i_0 small, and will generally be negligible. It has been possible to keep them small by including in c the main term of the precession to the mid-epoch. The convenient expression for ω was rendered possible by neglecting a small term in Ω which cannot amount to 0″·01 in two hundred years.

For reductions from 1950 to the year for which a, b, c, d are given we use

$$\begin{aligned}
\lambda_0 &= \lambda - a + b\cos(\lambda + c - a)\tan\beta - d\sin 2(\lambda + c - a)\tan^2\beta \\
\beta_0 &= \beta - b\sin(\lambda + c - a) - d\cos^2(\lambda + c - a)\tan\beta \\
\Omega_0 &= \Omega - a + b\sin(\Omega + c - a)\cot i + d\sin 2(\Omega + c - a)\cot^2 i \\
i_0 &= i - b\cos(\Omega + c - a) + d\sin^2(\Omega + c)\cot i \\
\omega_0 &= \omega - (\text{sum of 3rd and 4th terms in } \Omega_0)\sec i
\end{aligned}$$

Computation of Special Perturbations.—By supplying homogeneous planetary co-ordinates a great burden will be taken off the shoulders of the calculators who undertake to compute special perturbations. In fact, it is hoped that the provision of such material in an English ephemeris, together with an explanation in our own language of the method of using it, will give a stimulus to the work in this field, so much so indeed that we will not again have to tell the tale of a periodic comet being lost because the search ephemerides computed without perturbations failed to lead to its detection. To-day the amateur computer is with us, and that he is not to be despised is amply illustrated by the work now being done by the computing section of the British Astronomical Association. He has mastered such things as the prediction of occultations, the determination of orbits, and the computation of ephemerides from elliptical elements, because the data for these are available, together with good descriptions in English of the methods of working. The fact that, with few exceptions, he has not turned his hand to perturbations is due, not to any greater intrinsic difficulty in the computation, but to the double circumstance that much tedious preliminary hackwork has to be done before the required planetary co-ordinates are obtained, and that no up-to-date English description of suitable methods is available. The first obstacle will be completely removed if the suggestions here made are approved and can be carried out. Concerning the second difficulty, Watson's *Theoretical Astronomy* is out of print (and, according to the publishers, no immediate reprint is contemplated), and in any case has not been revised for sixty years. The best textbook is undoubtedly Bauschinger's *Bahnbestimmung der Himmelskörper* (Leipzig, 1906), also out of print. Professor Bauschinger has prepared a new edition, which the publishers unfortunately cannot afford to print at present. It should be possible, however, to describe and illustrate in a small compass

the best methods to be used with the data that it is hoped will be provided.

In considering the various methods that have been proposed for computing perturbations we must bear in mind that the method to provide for it is not necessarily the one most used at present, but rather the one that would be preferred were the necessary planetary data available. The three methods surviving to-day, and of which the first is perhaps the most widely used, are those of variation of elements, first introduced by Lagrange, perturbations of polar co-ordinates, introduced by Hansen, and perturbations of rectangular co-ordinates, due to Encke. More recently a valuable and important method was introduced by Cowell,* and successfully applied to Halley's comet. Here the ellipse is discarded and rectangular co-ordinates with origin at the C.G. built up by mechanical quadratures. The advantages of Cowell's method are :—

(1) The theory of the method is the simplest possible.
(2) Only three integrations are required as against six in the variation of elements.
(3) Terms depending on the higher powers of the masses are included without any indirect calculation.
(4) The advantages accruing from the use of the centre of gravity of the solar system can be realised.
(5) The numerical work is exceptionally straightforward, and is similar for each of the three co-ordinates. No trigonometrical tables are required, and the work lends itself admirably to the calculating machine.
(6) If, as is recommended, the heliocentric *equatorial* rectangular co-ordinates are used, an ephemeris can be prepared immediately from the material available when the perturbations are computed.

Where perturbations have to be computed over a long period the necessity for frequent rectification of the osculating ellipse is a serious disadvantage in Encke's method. To the writer it seems that, assuming planetary rectangular co-ordinates to be available, Cowell's method possesses decided advantages over any other, especially when considering the effects of all planets producing appreciable perturbations. Some objections which might be raised must be disposed of. Firstly, that there will be an accumulation of error due to uncertainty in the mean motion of the perturbed body. This is true of any method, and it remains to be demonstrated that Cowell's method is affected more by this uncertainty than other methods are. Secondly, that a very large number of decimals, perhaps nine or ten, have to be used in order to retain seven-figure accuracy over a long period of years. However, the bulk of the work is in the computation of the attractions, the greatest of which is that of the Sun, and even there not more than six (rarely

* P. H. Cowell and A. C. D. Crommelin, "Essay on the Return of Halley's Comet," *Publikationen der Astronomischen Gesellschaft*, No. 22 (1910). Also as Appendix to *Greenwich Observations* for the year 1909. See remarks on the method by J. Jackson in *M.N.R.A.S.*, **84**, 602.

seven) significant figures would be required. Ten significant figures would occur only in the double summation, a relatively small portion of the total labour.

But a real and serious difficulty arises when a comet is near perihelion (this difficulty does not occur with asteroids), because then, on account of the large attraction of the Sun, the interval must be diminished, perhaps even to one day, and more decimals retained. Following a hint given by Mr. G. Merton, it is suggested that one of the other methods should be used near perihelion. This is where Encke's method would work hand in hand with Cowell's. Using the given osculating elements, the comet would be carried forward by Encke's method to a point where Cowell's method could be used with an interval of, say, ten days. There is no work to be done in transferring from the former to the latter method; the computer simply ceases to use the one and commences to use the other.

The following table shows at a glance the data which would enable any of the above methods to be used. The co-ordinates x, y, z are referred to equatorial axes and to the C.G. of the solar system as origin, except in the last column, where the origin is at the Sun.

Planet, etc.	Interval in Days	No. of Decimals: λ and β in Degrees	r	$\frac{1}{r^3}$	x, y, z	$\frac{-20^2mk^2\times10^7}{r^3}$ (Also for y and z)
Sun	10	..	..	..	7	
C.G. of Sun and four inner planets	10	..	..	..	7	
Mercury	10	2	4	3	4	2 (10 days)
Venus	10	3	5	5	5	
Earth+Moon	10	3	5	5	5	
Mars	10	3	5	5	5	
Jupiter	10	4	5	8	5	2 (10 days)
Saturn	20	4	5	8	5	
Uranus	40	3	4	8	4	2 (40 days)
Neptune	40	3	4	8	4	

This would require eight pages per year. Where the interval is 10, 20, or 40 days the dates would be chosen so that the integral part of the Julian Day number is divisible by 10, 20, or 40.

Use of the Centre of Gravity of the Solar System.—In the following paragraphs it will be shown that the use of the C.G. of the solar system as origin is decidedly advantageous in the computation of special perturbations by rectangular co-ordinates. Let

x, y, z, r be the co-ordinates of the perturbed body, hereinafter called the comet

$x', y', z', \lambda', \beta', r'$ the co-ordinates of a typical perturbing planet of mass m'

$x_\odot, y_\odot, z_\odot$ the co-ordinates of the Sun referred to the C.G. of the solar system.

Only the equations in x will be written in order to save space.

We have

$$x_\odot = -\frac{\Sigma m' x'}{1+\Sigma m'} \quad . \quad . \quad . \quad . \quad (1)$$

where the origin of x' is the Sun. When the origin of x' is at the C.G.

$$\begin{aligned} x' &= r' \cos\beta' \cos\lambda' + x_\odot \\ y' &= r' \cos\beta' (\sin\lambda' \cos\epsilon - \tan\beta' \sin\epsilon) + y_\odot \\ z' &= r' \cos\beta' (\sin\lambda' \sin\epsilon + \tan\beta' \cos\epsilon) + z_\odot \end{aligned} \quad . \quad . \quad (2)$$

in which for 1950·0 $\sin\epsilon = 0{\cdot}39788119$
$\cos\epsilon = 0{\cdot}91743695$

The equations typified by

$$\frac{d^2x}{dt^2} + \frac{k^2 x}{r^3} = 0 \quad . \quad . \quad . \quad . \quad (3)$$

represent elliptical motion. When the perturbing action of planets is considered we have

$$\frac{d^2x}{dt^2} = -\frac{k^2 x}{r^3} + \Sigma m' k^2 \left(\frac{x'-x}{\rho^3} - \frac{x'}{r'^3}\right) \quad . \quad . \quad (4)$$

where

$$\rho^2 = (x'-x)^2 + (y'-y)^2 + (z'-z)^2 \quad . \quad . \quad (5)$$

If now we use the same notation, but transfer the origin to the C.G., equation (4) becomes

$$\frac{d^2x}{dt^2} = \frac{k^2(x_\odot - x)}{\rho_\odot{}^3} + \Sigma \frac{m' k^2 (x'-x)}{\rho^3} \quad . \quad . \quad . \quad (6)$$

which is the fundamental equation of Cowell's method. The terms $\Sigma \frac{m' k^2 x'}{r'^3}$, representing the attraction of the perturbing planets on the origin, have disappeared. Apart from the diminished numerical labour resulting from this, these terms change rapidly in the case of the inner planets, and their inclusion would necessitate a much shorter interval than can be used without them. Equation (6) is the acme of simplicity; it simply says that the comet moves under the combined attractions of the Sun and all the planets. When the comet is moderately distant from the Sun, and also from Mercury, Venus, Earth, and Mars, it will be expedient to replace k^2 in the first term by $k^2(1 + \text{sum of masses of four inner planets})$, and $x_\odot$ by $x_\square$, the co-ordinate of the C.G. of the Sun and four inner planets. It will be noted from the table above that it is proposed to provide this latter co-ordinate.

If Cowell's method were applied to minor planets, for which it seems to be very suitable when an accurate ephemeris is required, a 20-day interval would usually suffice, while a 10-day interval would satisfy the most rigorous requirements. In general, it would be sufficient to add the masses of Mercury, Venus, and the Earth to that of the Sun, and use the modified centre of gravity provided, for the effect of Mars on the co-ordinates of the C.G. can never exceed five units of the 7th decimal.

Unless unusual accuracy were required the perturbations of Mars, Jupiter, and Saturn only would be considered. The use of equatorial rather than ecliptic co-ordinates has the great advantage that an ephemeris can be made immediately from the co-ordinates resulting from the integrations, for if X, Y, Z are the Sun's co-ordinates (for equinox 1950·0) as tabulated

$$\begin{aligned} \Delta \cos\alpha \cos\delta &= x - x_\odot + X \\ \Delta \sin\alpha \cos\delta &= y - y_\odot + Y \\ \Delta \sin\delta &= z - z_\odot + Z \end{aligned} \quad . \quad . \quad . \quad (7)$$

Use of Encke's Method.—Encke uses the osculating elements to determine the co-ordinates x_0, y_0, z_0 (origin at the Sun) of the undisturbed elliptic motion. Calling the perturbations of these ξ, η, ζ, so that

$$x = x_0 + \xi \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Encke's working equation may be written

$$\frac{d^2\xi}{dt^2} = \Sigma m' k^2 \left(\frac{x' - x}{\rho^3} - \frac{x'}{r'^3} \right) + \frac{k^2}{r_0^3} \left(fq(x_0 + \xi) - \xi \right) \quad . \quad (9)$$

where ρ is the distance from the perturbing planet to the comet, q is defined by

$$r_0^2 q = (x_0 + \tfrac{1}{2}\xi)\xi + (y_0 + \tfrac{1}{2}\eta)\eta + (z_0 + \tfrac{1}{2}\zeta)\zeta \quad . \quad (10)$$

and f is a factor, not differing greatly from 3, that has been tabulated with argument q.

The chief disadvantages of this method are the rectification needed from time to time to prevent the second term in (9) from attaining the predominating influence, and the fact that the terms $\frac{m'k^2x'}{r'^3}$ change rapidly in the case of the inner planets and necessitate a short interval. The first of these is removed if we restrict the use of the method to the time when a comet is near perihelion, *i.e.* until such time as Cowell's method can be conveniently employed with an interval not less than ten days. Concerning the second, it was proposed by Bessel to refer the motion of a comet to the C.G. of the Sun and the disturbing planet. When the comet is distant from both this method is suitable. If we pursue this method to what might be deemed its logical conclusion, and refer the motion of the comet to the C.G. of the entire solar system, the term ξ in (10) is replaced by $\xi - x_\odot$, while (9) becomes

$$\frac{d^2\xi}{dt^2} = \Sigma \frac{m' k^2 (x' - x_0 - \xi)}{\rho^3} + \frac{k^2}{r_0^3} \left(fq(x_0 + \xi - x_\odot) - (\xi - x_\odot) \right) \quad (11)$$

While the terms $\frac{m'k^2x'}{r'^3}$ have disappeared, a term $\frac{k^2 x_\odot}{r_0^3}$ has been introduced. This represents a perturbation by the Sun, due to its displacement from the origin, and, for a comet near the Sun, would be disproportionately large, so that no advantage is offered by this transformation.

The conclusion is therefore reached that the use of Encke's method should be restricted to comparatively short periods, either for determining the perturbations required for a definitive orbit, or until a comet is, say, two units from the Sun, and that it is best used with the origin at the Sun. The second disadvantage mentioned in the preceding paragraph will not then be felt. Considerable help will be afforded, however, by tabulating the terms in question. It is at present suggested that instead of tabulating the individual values three sums should be given, one for the four inner planets, one for Jupiter and Saturn, and one for Uranus and Neptune. It may be recalled that in the *Publikationen der Astronomischen Gesellschaft*, I and VI, the values of x', $\frac{-40^2 m' k^2 x' 10^7}{r'^3}$, etc., were tabulated for Jupiter from 1770 to 1865, and for the other planets (except Mercury) from 1830 to 1865. Since the planetary co-ordinates tabulated will be referred to the C.G. (to cater for the more important method of Cowell) the working form of equation (9) will be

$$\frac{d^2\xi}{dt^2} = \Sigma \frac{m' k^2 (x' - x_\odot - x)}{\rho^3} + \frac{k^2}{r_0^3}\left(fq(x - \xi)\right) + \text{tabulated terms} . \qquad (12)$$

Log f is given in treatises on theoretical astronomy; a table of the natural values of f should be prepared. If the use of the method is limited to the cases mentioned above, the range of this table would be much smaller than that of the present tables of log$f$.

Conclusion.—We have tried to show that Encke's method may be used to advantage while a comet is near perihelion, and Cowell's method at other times; also that the latter would be specially appropriate for minor planets of which complete perturbations are desired. It presents the dynamical problem in its simplest possible aspect, while the numerical work is of the most uniform and straightforward nature. We believe that the publication by the *N.A.* of the co-ordinates required for its application would give a great stimulus to its use.

Notes added in Press.—After sending the above paper to press it became evident that it would really be better to give the planetary co-ordinates x', y', z' with origin at the Sun. Apart from the incidental consideration that they will then be in the form required for use in Encke's method, the origin in Cowell's method must always be at the C.G. of the Sun and the planets whose perturbations are being considered. If the perturbations of Uranus and Neptune are omitted, these planets must also be omitted when forming the co-ordinates of the origin referred to the Sun. It seems advisable, therefore, to tabulate, with origin at the Sun

x' co-ordinate of perturbing planets
x_0 co-ordinate of C.G. of Sun and all planets
x_1 co-ordinate of C.G. of Sun and all planets, except Uranus and Neptune
$x_\triangle$ co-ordinate of C.G. of Sun and three inner planets
$x_\square$ co-ordinate of C.G. of Sun and four inner planets.

If, then, x is the co-ordinate of the comet referred to the origin, and $x_\odot$ its co-ordinate referred to the Sun, Cowell's equations become

$$x_\odot = x + x_0 \quad \text{if using all planets}$$

or $$x_\odot = x + x_1 \quad \text{if omitting Uranus and Neptune}$$

$$\frac{d^2x}{dt^2} = \frac{-k^2x_\odot}{r_\odot{}^3} + \Sigma\frac{m'k^2(x'-x_\odot)}{\rho^3}$$

If some of the inner planets, say all four, are allowed for by the use of an enhanced Sun, then, since the maximum value of $x_\square$ is only half a unit in the fifth decimal and no x' is given to more than five decimals, we may write

$$x_\odot = x + x_0 - x_\square$$

and then

$$\frac{d^2x}{dt^2} = \frac{-k^2(1+m_\square)x_\odot}{r_\odot{}^3} + \Sigma\frac{m'k^2(x'-x_\odot)}{\rho^3}$$

The following liberties and restrictions then occur :—

(1) Uranus and Neptune can be included or omitted at will. They may not be separated.
(2) All the planets to Saturn inclusive must be included.
(3) Mercury alone (since its effect on the C.G. can never exceed one unit of the seventh decimal), or Mercury, Venus, Earth, or all four inner planets, can be allowed for by the use of an enhanced Sun.

It is believed that the restrictions will not present any inconvenience in practice.

Instead of publishing the planetary data in the *N.A.* year by year it would probably be better to issue a separate supplement, say about 1929, covering (if possible) the 40 years 1900–39.

Observations of V Cassiopeiæ (Ch. 8324) in 1921–26.

By A. N. Brown, M.A.

The last series of my observations of this long-period variable appeared in *Monthly Notices*, **81**, 469–474, the result of another five years' work being represented by the 258 observations printed below. The details regarding the maxima and minima which follow are derived from the light curves drawn from these observations.

Maxima.			Minima.		
Date.	Julian Date.	Mag.	Date.	Julian Date.	Mag.
1921 June 28	2869	8·1	1921 Oct. 21	2984	12·8
1922 Feb. 12	3098	8·2	1922 June 9	3215	12·7
1922 Sept. 29	3327	8·1	1923 Jan. 29	3449	12·3
1923 May 21	3561	8·1	1923 Sept. 12	3675	12·4
1923 Dec. 23	3777	8·1	1924 May 3	3909	12·3
1924 Aug. 5	4003	7·9	1924 Nov. 27	4117	11·9
1925 Mar. 19	4229	7·6	1925 July 20	4352	12·3
1925 Oct. 25	4449	7·6	1926 Mar. 16	4591	12·6

It may be noticed that the magnitude assigned for the third of these maxima differs slightly from that actually recorded for 1922 Sept. 29. The reason is that the observation in question as well as that which immediately follows it having been made in moonlight was given less weight than those preceding and following, which appear to indicate the magnitude entered in the above table for the date named.

A careful analysis shows that, while the mean period of the variable since the observations began in 1909 to the present year is 228·2 days, the period has been gradually lengthening during this time. Until 1916 it was 227·1 days, from 1916–21 229 days, and for the present series of observations it is 231·7 days. Minima as well as maxima have been used in deducing these periods.

Usually the rise to maximum, for which the average time from 1909–25 is 107·8 days, is more rapid than the fall to minimum (121 days), but this is not invariably the case. On four occasions since 1909 the fall was completed in a rather shorter time than the rise, and once (in 1913) the rise and fall were of equal duration.

In making the comparisons given in the third column below the "fractional" method has most usually been employed. Thus the first observation of this series is intended to indicate that the variable was considered exactly intermediate in brightness between the two comparison stars used, the second that it was $\frac{3}{4}$ of the whole light interval from the brighter star, and so on. But sometimes, as on 1921 Sept. 7, estimates were made directly in tenths of a magnitude, this particular observation showing that the variable appeared to be ·1 mag. fainter than the comparison star 69. When both methods were used on the same occasion, the mean is recorded in the fourth column, equal weight being given to each result. The same telescopes have been used as previously, namely 5-inch and 3-inch refractors with powers varying from 25 to 150.

The Observations.

Date.	Julian Date.	Comparisons.	Ded. Mag.	Date.	Julian Date.	Comparisons.	Ded. Mag.
1921.				1921.			
Apr. 14	2794	51(1)v(1)55	11·3	July 8	2879	4(2)v(1)5	8·1
27	2807	32(3)v(1)43	10·8	12	2883	4(2)v(1)5	8·1
30	2810	32(3)v(1)43	10·8	20	2891	5(2)v(1)7	8·5
May 8	2818	32(1)v(1)43	10·6	25	2896	7(2)v(1)11	8·9
15	2825	22(3)v(1)32	10·2	Aug. 8	2910	22(2)v(1)32	10·2
21	2831	15(1)v(1)17	9·6	25	2927	51(1)v(1)55	11·3
June 6	2847	5(1)v(2)7	8·4	Sept. 3	2936	64(2)v(1)69	11·9
11	2852	4(3)v(1)5	8·2	7	2940	69−1	12·1
15	2856	4(5)v(2)5	8·1	21	2954	69(3)v(1)73	12·2
17	2858	4(2)v(1)5	8·1	25	2958	73+0·5	12·3
23	2864	4(2)v(1)5	8·1	29	2962	73−1, =75	12·4
28	2869	4(2)v(1)5	8·1	Oct. 1	2964	75−1	12·5
July 5	2876	4(2)v(1)5	8·1	4	2967	75−1	12·5

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1921.			
Oct. 8	2971	75−2	12·6
23	2986	75−4	12·8
30	2993	75−3	12·7
Nov. 6	3000	75−3	12·7
23	3017	64(3)v(1)69	12·0
26	3020	64−2, =69	12·0
Dec. 20	3044	43(2)v(3)51	11·0
25	3049	32(2)v(1)43	10·7
28	3052	32(1)v(3)43	10·5
31	3055	32(1)v(2)34	10·5
1922.			
Jan. 3	3058	22−2, 32+2	10·1
10	3065	13(2)v(1)22, 17(1)v(1)21	9·7
16	3071	15(2)v(1)17	9·6
20	3075	9(3)v(1)14	9·3
Feb. 1	3087	4−5, 5+1	8·2
9	3095	4(3)v(1)5	8·2
11	3097	4(3)v(1)5	8·2
13	3099	4(4)v(1)5	8·2
17	3103	4(3)v(1)5	8·2
20	3106	4(3)v(1)5	8·2
24	3110	5+0·5	8·3
Mar. 2	3116	5(3)v(1)7	8·5
7	3121	7(1)v(2)11	8·8
12	3126	7(3)v(1)11, =9	8·9
20	3134	=17	9·6
27	3141	21−0·5	9·9
Apr. 1	3146	22(1)v(1)34	10·2
19	3164	43(2)v(1)51	11·1
24	3169	51(1)v(2)55	11·3
May 2	3177	64(3)v(1)69	12·0
14	3189	69(3)v(1)73	12·2
19	3194	69(3)v(1)73	12·2
23	3198	75−2	12·6
27	3202	75−2	12·6
June 1	3207	75−3	12·7
July 18	3254	43(2)v(1)51	11·1
July 28	3264	32(1)v(3)43	10·5
Aug. 20	3287	9(2)v(1)14	9·2
23	3290	9(2)v(1)14	9·2
31	3298	5(1)v(1)7	8·5
Sept. 9	3307	4(3)v(1)5	8·2
15	3313	4(3)v(2)5	8·1
20	3318	4(3)v(2)5	8·1
24	3322	4(2)v(1)5	8·1
29	3327	4(3)v(1)5	8·2
Oct. 6	3334	4(3)v(1)5	8·2
12	3340	4(2)v(1)5	8·1
15	3343	4(3)v(1)5	8·2
22	3350	4(4)v(1)5	8·2
28	3356	5(1)v(2)7	8·4
Nov. 13	3372	11(3)v(1)15	9·4
15	3374	15(1)v(2)17	9·5
18	3377	15(1)v(1)17	9·6
24	3383	17(1)v(1)22	9·8
Dec. 6	3395	22(4)v(1)34	10·3
20	3409	43(4)v(1)51	11·1
26	3415	55−1, 64+1	11·6
1923.			
Jan. 8	3428	64−2	12·0
11	3431	64(4)v(1)69	12·0
14	3434	=69	12·0
25	3445	69−3, 73+1	12·3
Feb. 4	3455	69(3)v(1)73	12·2
11	3462	69(2)v(1)73	12·2
Mar. 4	3483	32(3)v(1)43	10·8
17	3496	17(1)v(1)22	9·8
26	3505	7(5)v(1)11	9·0
Apr. 3	3513	7(2)v(3)11	8·8
8	3518	5−2·5, 7+0·5	8·6
13	3523	5(3)v(1)7	8·5
20	3530	5(3)v(1)7	8·5
26	3536	5(3)v(2)7	8·5
May 5	3545	4(3)v(1)5	8·2
16	3556	4(4)v(1)5	8·2

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1923.			
May 21	3561	4(2)v(1)5	8·1
25	3565	4(3)v(1)5	8·2
June 11	3582	=5	8·3
July 4	3605	11(3)v(1)15	9·4
8	3609	15−1, =16, 19+2	9·5
20	3621	22(4)v(1)34	10·3
24	3625	34(1)v(2)43	10·6
28	3629	32(5)v(1)43	10·8
Aug. 4	3636	=51	11·2
8	3640	=55	11·4
11	3643	55(2)v(1)64	11·7
13	3645	55(3)v(1)64	11·7
30	3662	69(3)v(1)73	12·2
Sept. 5	3668	73−1	12·4
8	3671	73−1	12·4
11	3674	73−1	12·4
15	3678	=75	12·4
29	3692	69−3, 75+1	12·3
Oct. 4	3697	69(2)v(1)73	12·2
14	3707	55(2)v(1)64	11·7
22	3715	43(2)v(1)51	11·1
28	3721	32(2)v(1)43, 34−4	10·8
Nov. 7	3731	22(1)v(4)34	10·0
10	3734	17−2, 22+1	9·8
18	3742	15(1)v(1)17	9·6
30	3754	7(1)v(3)11	8·7
Dec. 3	3757	5(3)v(1)7	8·5
15	3769	4−4, 5+2	8·1
18	3772	4(7)v(5)5	8·1
20	3774	5+2	8·1
23	3777	4−4, 5+2	8·1
1924.			
Jan. 2	3787	4−4, 5+2	8·1
6	3791	4−5, 5+1	8·2
20	3805	5(1)v(1)7	8·5
25	3810	5(1)v(1)7	8·5
29	3814	7(3)v(2)11	8·9
Feb. 1	3817	7(3)v(1)11	9·0
1924.			
Feb. 16	3832	15(1)v(1)17	9·6
24	3840	22(3)v(2)34	10·2
Mar. 2	3847	32−1, 34+1	10·4
4	3849	32(2)v(1)43	10·7
7	3852	34(2)v(1)43	10·7
10	3855	43+1	10·8
29	3874	51(4)v(1)64	11·7
Apr. 1	3877	55(3)v(1)64	11·7
4	3880	64(2)v(1)69	11·9
7	3883	64(3)v(1)69	12·0
May 1	3907	69(4)v(1)73, 75+1	12·3
7	3913	69(4)v(1)73, 75+1	12·3
27	3933	55(1)v(1)64	11·6
June 13	3950	17(1)v(2)22	9·7
20	3957	9(3)v(2)14	9·2
July 10	3977	5(2)v(1)7	8·5
13	3980	5(1)v(1)7	8·5
19	3986	4−5, 5+1	8·2
24	3991	4−3, 5+3	8·0
29	3996	4(1)v(1)5	8·0
Aug. 2	4000	4−2, 5+4	7·9
5	4003	4−2, 5+4	7·9
7	4005	4−2, 5+4	7·9
12	4010	4(3)v(2)5	8·1
19	4017	4−4, 5+2	8·1
25	4023	4−4, 5+2	8·1
31	4029	5(1)v(1)7	8·5
Sept. 7	4036	5(1)v(1)7	8·5
14	4043	7(2)v(3)11	8·8
22	4051	15(1)v(2)22	9·6
27	4056	22(1)v(4)34	10·0
Oct. 8	4067	22(2)v(1)32	10·2
19	4078	32(2)v(1)43	10·7
23	4082	34(3)v(1)43	10·8
Nov. 3	4093	43(3)v(1)51	11·1
17	4107	55(2)v(3)64	11·6
27	4117	64(1)v(2)69	11·9
Dec. 6	4126	51(2)v(1)55	11·3

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1924.			
Dec. 14	4134	51(2)v(1)55	11·3
26	4146	43(2)v(1)51	11·1
30	4150	34−5, 43+1	10·9
1925.			
Jan. 4	4155	34(2)v(1)43	10·7
6	4157	34(1)v(2)43	10·6
9	4160	32−2, 34−1	10·5
15	4166	32(1)v(1)34	10·4
22	4173	32+2, 34+3	10·1
Feb. 1	4183	17(1)v(1)21	9·7
6	4188	14−1, 15+1, 17+2	9·4
15	4197	7(1)v(1)11	8·9
17	4199	7(1)v(3)9	8·7
23	4205	5−1, 7+3	8·4
28	4210	4−4, 5+2	8·1
Mar. 2	4212	4−3, 5+3	8·0
8	4218	4−3, 5+3	8·0
12	4222	4−1, 5+5	7·8
18	4228	4 + 1	7·6
21	4231	4+1	7·6
28	4238	4−1, 5+5	7·8
Apr. 1	4242	4−1·5	7·9
3	4244	4−2, 5+4	7·9
8	4249	4(2)v(1)5	8·1
12	4253	4−4, 5+2·5	8·1
21	4262	5(3)v(1)7	8·5
28	4269	7(3)v(1)11	9·0
May 12	4283	22(1)v(4)34	10·0
20	4291	22−4, 32+1, 34+2	10·2
25	4296	32(1)v(2)43, 34−1	10·5
28	4299	32(2)v(1)43	10·7
June 1	4303	32(3)v(1)43	10·8
8	4310	43(3)v(1)51	11·1
11	4313	43(4)v(1)51	11·1
17	4319	51−3, 55−1, 64+3	11·5
29	4331	64(1)v(2)69	11·9
July 7	4339	64(3)v(1)69	12·0
11	4343	=69	12·0
15	4347	69(2)v(1)73	12·2
18	4350	=73	12·3
Aug. 2	4365	64(3)v(1)69	12·0
7	4370	69+1	11·9
16	4379	=64	11·8
29	4392	32−1, =34	10·4
Sept. 4	4398	22−3, 32+1, 34+2	10·2
1925.			
Sept. 10	4404	22(1)v(2)32	10·0
14	4408	15(1)v(1)22	9·7
18	4412	14+1, 15+2	9·3
23	4417	8−1, 9(3)v(1)14, 11+2	9·1
26	4420	5(2)v(1)7	8·5
Oct. 3	4427	4(4)v(1)5	8·2
8	4432	4−4. 5+2	8·1
13	4437	4−2, 5+4	7·9
17	4441	4−4, 5+2	8·1
23	4447	4+1	7·6
27	4451	4+1	7·6
Nov. 4	4459	=4	7·7
8	4463	4(1)v(1)5	8·0
12	4467	4−4, 5+2	8·1
16	4471	4−4, 5+2	8·1
26	4481	=5, 7 + 2·5	8·3
Dec. 5	4490	5(4)v(1)7	8·5
9	4494	7(4)v(1)11, 9(1)v(4)14	9·0
17	4502	17−2, =21, 22+1	9·8
31	4516	=32	10·3
1926.			
Jan. 3	4519	32−3, 34−2, 43+3	10·6
6	4522	34(4)v(1)43	10·8
11	4527	43(3)v(2)51	11·1
16	4532	51+1	11·1
19	4535	51(1)v(2)55	11·3
Feb. 13	4560	64−3, =69	12·1
15	4562	69(3)v(2)73	12·2
Mar. 10	4585	73−3, 75−2	12·6
13	4588	75−2	12·6
15	4590	73−3, 75−2	12·6
21	4596	75−2	12·6
30	4605	75−1·5	12·6
Apr. 9	4615	51(1)v(2)55	11·3
11	4617	43(2)v(1)51	11·1
15	4621	=43	10·9
17	4623	32(1)v(1)43, 34−2	10·6
May 3	4639	9(2)v(1)14	9·2
9	4645	9(1)v(1)11	9·0
13	4649	7(1)v(1)11	8·9
15	4651	7(2)v(3)11	8·8

The Comparison Stars Used.

Hagen's Number.	H.P. Magnitude.	Hagen's Number.	H.P. Magnitude.	Hagen's Number.	H.P. Magnitude.
4	7·7	15	9·5	43	10·9
5	8·3	16	9·5	51	11·2
7	8·6	17	9·6	55	11·4
8	8·8	19	9·7	64	11·8
9	8·9	21	9·8	69	12·0
11	9·1	22	9·9	73	12·3
13	9·3	32	10·3	75	12·4
14	9·4	34	10·4		

Brackenhurst,
Bucklebury Common, Berks:
1926 *May* 20.

Note on the Nova in N.G.C. 4303 (*M.* 61).

(*Communicated by the Astronomer Royal.*)

Photographs of the Nova in N.G.C. 4303 were obtained with the Astrographic Refractor on the following dates: 1926 May 15, 16, and 31.

In the period during which the Nova was under observation the photographic magnitude decreased by about 0·3 mag. by comparison with a small star ("A") immediately following the nebula, and possibly connected with it. From being distinctly brighter than star "A" on May 15, the Nova had become slightly fainter than star "A" on May 31. The photographic magnitude is about 13, but until the magnitudes of comparison stars have been obtained it is not possible to make a determination with accuracy. The positions of the Nova and of star "A" with reference to the nucleus of the nebula, as deduced from the plates, are:

	Angle.	Dist.
	°	″
Nova	350·0	67·3
Star "A"	256·5	72·0

New Nebulæ shown on Franklin-Adams Chart Plates.

By P. J. Melotte. (Plate 15.)

The following nebulæ, which do not appear to have been recorded elsewhere, were noted when drawing up the Catalogue of Star Clusters shown on Franklin-Adams Chart Plates. They were also independently noted by Dr. Lundmark when examining the same plates. The positions given are for the epoch 1900·0.

Extended Nebula in Vela.

Centre (1900·0) R.A. 8^h 31^m. Dec. $-44°$ 20′.
(1 réseau interval is equal to 1°.)

R.A. $5^h\ 15^m.9$ Dec. $+8°\ 13'$

Diffuse nebula on *south preceding* side of B.D. $+8°$, 933 (mag. 6·5) with which it appears to be associated. The brightest part is roughly pear-shaped, with the pointed end in the direction of the star. There is a noticeable falling-off in star density in the area surrounding the nebula, particularly to the south-west, where there is a strongly marked vacant space over one degree square. A fifteenth magnitude star is near the centre of the brightest part of the nebula, approximately 5′ west and 4′ south of B.D. 8°, 933. The nebula is well shown on F.A. Chart 121, where it falls near to the edge of the plate, and it also appears on Chart 97, but somewhat fainter. The fact that this nebula has not been noted before might be explained if it proved to be variable.

R.A. $8^h\ 30^m$. Dec. $-44°\ 0'$. (See Plate 15.)

Large extended nebula in Vela. The nebula lies between R.A. $8^h\ 22^m$ and $8^h\ 39^m$, and extends from Dec.$-42°\ 0'$ to $-46°\ 30'$. It consists of a number of bright nebulous streaks, some of the brightest of which outline an area, seven square degrees in extent, roughly oval in form. The brightest part lies between the two stars at $\alpha\ 8^h\ 26^m.0$, $\delta -44°\ 23'$ (mag. 5·1) and $\alpha\ 8^h\ 34^m.2$, $\delta -42°\ 38'$ (mag. 4·1), *e* Velorum, where there are several branches of the nebula. There is much bright diffuse nebula to the south of the area indicated by the limits given above, but its appearance on the plate is confused by the densely crowded star-field. A number of stars in the region are nebulous or show traces of nebulosity near them, of which the following may be noted:—

Mag.	R.A. h m	Dec. ° ′	No. °	Mag.	R.A. h m	Dec. ° ′	No. °
6·4	8 25·7	−43 50	−43, 2636	5·5	8 37·3	−46 17	−46, 2805
8·2	8 26·0	−41 11	−41, 2599	6·0	8 37·9	−46 57	−46, 2819
5·9	8 26·0	−44 23	−44, 2667	6·0	8 38·5	−45 3	−44, 2936
8·0	8 26·2	−43 46	−43, 2645	7·9	8 40·9	−40 55	−40, 2824
6·4	8 25·9	−47 36	−47, 2296	4·8	8 42·5	−45 40	−45, 3014
7·3	8 28·0	−47 32	−47, 2348	5·2	8 34·8	−42 43	−42, 2754
9·4	8 31·8	−40 19	−40, 2666 *				

* *N.G.C.* 2626.

The magnitudes and numbers are from the Cape Photographic Durchmusterung.

R.A. $11^h\ 5·3^m$. Dec. $-77°\ 7'$.

Nebulous star surrounded with bright nebula (C.P.D. $-76°$, 654, mag. 8·5). Falls near centre of a vacant area 2·2 square degrees in extent. There is a small patch of nebula near a twelfth magnitude star at $\alpha\ 11^h\ 3^m$, $\delta -76°\ 48'$. A bright nebula (*H.A.*, lx. 6, p. 161, No. 352) is at the northern end of the vacant area. The obscuration is well defined amongst the surrounding star-field and contains some traces of faint nebulosity in addition to those given above.

R.A. $23^h\ 56^m.9$. Dec. $-15°\ 59'$.

Faint nebula, $10' \times 2'.5$. A faint star (mag. 15·5) is at the centre, and another star (mag. 16) falls at the southern edge. The nebula is condensed towards the centre, and the southern half appears brighter than the northern. The star distribution in the surrounding area is irregular, the region on the *following* side being almost devoid of stars. The appearance of this nebula and of Barnard's Nebula, *N.G.C.* 6822, is strikingly similar on the plates. The galactic co-ordinates are: Long. 46°, Lat. −74°.

An Ephemeris of the Geminid Radiant-point.

By A. King.

The first person to call special attention to the existence of the Geminid shower of meteors was Mr. R. P. Greg. Observing at Manchester on 1862 December 10–12, he fixed the radiant at 100°+33°, from many paths. This observation was confirmed by Messrs. Marsh and Twining in the United States in the same year.* The position, although undoubtedly in error as regards R.A., appears to have been the first definitive determination of the radiant-point of the shower.

A glance at the observedpositions of the Geminid Shower, LXXXVIII. of Mr. W. F. Denning's admirable "General Catalogue," shows at once that they cannot all refer to the same stream. To mention declination alone, we have places varying between the limits +27° and +40°. The truth is that, contemporary with the Geminids, there are several active radiants lying close to the line of march of the centre of the main stream, and furnishing shooting stars very similar in appearance to the members of that shower. It is therefore hard at times to distinguish between true Geminids and meteors belonging to these secondary streams. This must have led to the publication of incorrect radiant-places and to the inclusion of positions of these minor showers in the above-mentioned list. It is, of course, apparent that these debatable determinations were included in this section of the "General Catalogue" because they were considered as Geminid centres by the observers themselves, the section merely aiming at giving a summary of the work done on the shower up to the time of compilation.

All this makes a critical examination ot the material a somewhat nice matter. In recent years, however, some very good determinations of the radiant-positions of the Geminids have been made, so that the time seemed opportune for a more precise statement of the places of the shifting radiant than has heretofore appeared.

The motion of the radiant was first detected by Mr. Denning in 1885, and he published a rough ephemeris in 1923.†

The list which follows contains, it is believed, practically all the determinations of the positions of the Geminid radiant which have yet been published. Though a few obviously erratic centres have been weeded out, it is pretty certain that some of the incorrect places above referred to have been included, but in the nature of the case this could hardly be avoided.

* Prof. A. S. Herschel, in *M.N.*, 1865, p. 63. † *M.N.*, Nov. 1923, p. 46.

Date.		Position. α	Position. δ.	No. of Meteors.	Observer.
13	Nov. 27–28	98°	+34°*	5	W.F.D.
85	29–30	98	+33	8	W.F.D.
arious years	Dec. 1–3	100	+33	4	W.F.D.
16	3	102	+32	6	W.F.D.
21	3	100	+32	5	A.K.
85	1–4	105	+34	8	W.F.D.
16	4	104	+32	4	W.F.D.
12	6	104½	+33*	..	W.F.D.
arious years	6–7	105	+32	12	W.F.D.
76	6–8	103	+31	20	W.F.D.
85	7	107	+33	5	W.F.D.
14	8	105	+31	4	W.F.D.
18	8	107	+32	6	W.F.D.
77	6–10	109	+33	9	W.F.D.
arious years	9	108	+34	4	A.K.
17	9	108	+32	12	W.F.D.
18	9	108	+33	6	W.F.D.
21	9	108	+32½	8	W.F.D.
77	9	108	+28	20	H.C.
92	9	109	+34	40	H.C.
96	9	111	+34	..	H.C.
15	10	109·6	+33·2	11	C.P.O.
15	10	111	+33	5	R.L.
17	10	110	+34	18	F.W.
20	10	109·1	+33·7	11	G.S.
22	10	110·7	+32½	3	R.M.D. –A.K.
25	10	110½	+32½	14	A.K.
87	10	113	+34½	10	D.B.
14	11	112·3	+32·1	4–9	N.B.
15	11	110½	+34	20	F.W.
17	11	111½	+33	14	F.W.
25	11	111½	+32½	16	A.K.
96	9–12	112	+32	132	H.C.
96	11	110	+34	..	H.C.
96	11	111	+33	17	E.R.B.
00	12	111	+31½	6	C.L.B.
03	11–13	110	+36	15	A.S.H.
14	12	109	+33	6	W.F.D.
14	12	109·1	+31·9	21+	N.B.

Date.		Position. α.	Position. δ.	No. of Meteors.	Observer.
1915	Dec. 12	110°	+33°	12	W.F.D.
1915	12	111½	+32½	10	F.W.
1917	12	111·9	+34·7	7+	C.P.O.
1917	12	112	+33	12	W.F.D.
1917	12	112	+33	17	F.W.
1920	12	111·0	+30·4	29	G.S.
1922	12	111½	+32	60	R.M.D. –A.K.
1925	12	112½	+32¼	36	A.K.
1925	12	113·2	+31·3	Many	R.M.D.
1896	11–12	112	+32	..	Hardcastle –A.S.H.
1866	12	112	+34	Many	W.H.W.
1872	12	112	+30	7	Jackson.
1876	12	109	+33	24	W.F.D.
1892	12	107	+34	6	W.F.D.
1892	12	107	+32½	17	E.R.B.
1892	12	109	+34	38	H.C.
1892	12	109	+30	5	W.H.M.
1892	12	112	+38	6	A.S.H.
1894	12	110	+34	14	W.H.M.
1898	12	108	+33	20	E.R.B.
1898	12	111½	+33	5	A.K.
1900	13	113	+32	..	T.H.A.
1906	13	111½	+33½	6	A.K.
1915	13	112	+35	12	F.W.
1917	13	113	+34·3	12	C.P.O.
1919	13	111	+32	4	A.K.
1919	13	114	+33½	18	W.F.D.
1922	13	113½	+31½	11	A.G.C.
1922	13	115	+32½	6	A.K.
1923	13	113½	+32½	25	A.K.
1925	13	113½	+32	14	A.K.
1923	14	115	+32	5	A.K.
1925	14	114	+32	4	A.K.
1914	15	114	+33	6	W.F.D.
1922	15	113½	+31	30	R.M.D. –A.K.
1922	17	114	+31	5	R.M.D. –A.K.

* Suspected.

In the column headed "Observer," "R.M.D.–A.K." refers to positions derived by the present writer from Mr. Dole's charts of his observations in 1922. In 1925 Mr. Dole gave the position for Dec. 12 as $\frac{1}{5}$ way from α to β Geminorum, and did not state the exact number of Geminids.

The initials of observers stand for :—

T.H.A.	T. H. Astbury, Wallingford.
E.R.B.	E. R. Blakeley, Dewsbury.
D.B.	David Booth, Leeds.
C.L.B.	C. L. Brook, Meltham, Huddersfield.
N.B.	*Nels Bruseth, Los Angeles, and Silvana (Wash.), U.S.A.
A.G.C.	A. Grace Cook, Stowmarket, Suffolk.
H.C.	H. Corder, Writtle (Chelmsford) and Bridgwater.
W.F.D.	W. F. Denning, Bristol.
R.M.D.	R. M. Dole, East Lansing, Mich., U.S.A.
A.S.H.	A. S. Herschel, Slough.
A.K.	A. King, Leicester and Ashby.
R.L.	*Raymond Lambert, Newark, N.J., U.S.A.
W.H.M.	W. H. Milligan, Belfast.
C.P.O.	*C. P. Olivier, Virginia, U.S.A.
G.S.	G. Shajn, Perm, Russia.
F.W.	Fiammetta Wilson, Totteridge, Herts.
W.H.W.	W. H. Wood, Birmingham.

The observed positions were next corrected for epoch (*i.e.* position of the year of observation in the cycle of years beginning with leap-year) and for precession, bringing them up to 1928. The results were then averaged as under :—

Date.	Av. Position, 1928.		No. of Radiants.	Date.	Av. Position, 1928.		No. of Radiants.
	°	°			°	°	
Nov. 27·5	98·5	+34·0	1	Dec. 10	110·4	+32·7	9
30·5	99·0	+33·0	1	11	112·6	+33·2	5
Dec. 2	100·5	+33·0	1	11·5	112·5	+31·9	1
3	101·3	+32·0	2	12	111·7	+32·7	15
3·5	106·0	+34·0	1	12·5	112·5	+31·9	1
4	104·2	+32·0	1	13	112·1	+33·0	21
6	104·7	+33·0	1	14	115·1	+32·0	2
6·5	105·2	+32·0	1	15	114·5	+32·0	2
8	106·3	+31·7	4	17	114·7	+31·0	1
9	108·9	+32·9	5				

In obtaining these positions, radiants determined before 1900 were advanced a day, to allow for the lapse of the leap-year day in 1900. This explains the place of such radiants in the list above. Centres found by combining meteors over more than one day were considered as on

* The determinations of these observers were taken from *Pub. Leander M'Cormick Obs.*, vol. ii, part 7.

the mean date, but no position covering more than four days was admitted.

The writer's own determinations, treated likewise, are :—

Date.	Av. Position, 1928.		No. of Radiants.	Date.	Av. Position, 1928.		No. of Radiants.
	°	°			°	°	
Dec. 3	100·4	+32·0	1	Dec. 12	112·8	+32·2	1
9	109·0	+34·0	1	13	113·3	+32·5	6
10	110·8	+32·5	1	14	115·1	+32·0	2
11	111·8	+32·5	1				

The two series of positions thus well demonstrate the eastward drift of the radiant. The following ephemeris was based on these positions :—

Ephemeris of the Geminid Radiant-point, 1928.

	α.	δ.		α.	δ.
	°	°		°	°
Nov. 25	91·3	+33·8	Dec. 7	106·4	+33·0
26	92·5	+33·7	8	107·7	+32·9
27	93·8	+33·7	9	108·9	+32·8
28	95·1	+33·6	10	110·1	+32·7
29	96·4	+33·6	11	111·3	+32·6
30	97·7	+33·5	12	112·5	+32·5
Dec. 1	98·9	+33·5	13	113·7	+32·3
2	100·2	+33·4	14	114·9	+32·1
3	101·4	+33·4	15	116·1	+31·9
4	102·7	+33·3	16	117·2	+31·7
5	104·0	+33·2	17	118·3	+31·5
6	105·2	+33·1			

These places may be taken as for Greenwich midnight terminating the corresponding date. There is not, at present, much evidence of the activity of the shower before November 30 (though Mr. Denning states * that he has seen Geminids from November 25 onwards) or after December 15.

It is hoped that exact observations in coming years will demonstrate the accuracy or otherwise of this ephemeris.

In conclusion, the writer wishes gratefully to acknowledge the valuable and untiring help of Mr W. F. Denning in placing at his disposal material from which many of the data on which the present paper is based were obtained.

Ashby, Scunthorpe, Lincs :
1926 *June* 5.

* *M.N.*, Nov. 1923, pp. 46, 47.

Fifth Note on W. Herschel's Extensive Nebulous Fields.

By J. G. Hagen, S.J., Assoc.R.A.S.

The present set of Herschel's nebulous regions is remarkable for the high galactic latitude of some of them and for the number of nebulæ seen or suspected in them.

1. Regarding Table I., it is only to be remembered that the $\Delta\alpha$ and $\Delta\delta$ of the guiding stars are counted in the sense : star−centre.

TABLE I.

No.	Centre (1925). h m ° ′	Guiding Star (1925).	h m ° ′	Mag.	$\Delta\alpha$. ′	$\Delta\delta$. ′
32	9 39·0+71 6	Berlin C 1344	9 27·9+70 9	4·8	−58	−57
		N.G.C. 3034	9 49·8+70 3	..	+57	−63
33	10 13·0− 9 11	Wien Ott. 3966	10 14·3− 8 41	6·7	+19	+30
34	10 24·0+51 24	Cambridge U.S. 3649	10 25·6+50 57	6·6	+15	−27
35	10 42·7+62 37	Helsingfors 6494	10 45·2+62 7	8·4	+17	−30
36	11 6·0+62 36	Helsingfors 6607	10 59·1+62 9	2·0	−47	−27
37	12 3·4+30 29	Leiden 4600	12 4·4+30 42	7·2	+14	+13
38	12 14·0+30 29	Leiden 4642	12 14·7+30 40	6·5	+10	+11
39	13 13·5+34 0	Leiden 4875	13 15·0+34 30	6·2	+18	+30
40	14 3·4+34 1	Leiden 5089	14 2·7+34 45	8·4	− 8	+44
41	15 10·7+18 51	Berlin A 5473	15 8·6+19 15	5·8	−28	+24

In Table II. Herschel's estimates of extension along the Parallel and the Meridian are again divided by two in order to count them from the centre of the field. In no case did Herschel assign a limit beyond which the nebulosity could not be traced. Those numbered 32, 33, and 34 he followed over four degrees in one or the other of the two directions. It will be noticed that the density of these fields is rather strong, frequently V and nowhere less than III to IV, except within a small gap on No. 38.

TABLE II.

No.	Parallel. ′	Meridian. ′	Herschel's Description.	Gal. Lat. °
32	12	122	Much affected with very faint whitish nebulosity	+39
33	119	69	Very faint whitish nebulosity	+39
34	12	125	Much affected	+54
35	15	98	Affected with very faint nebulosity	+49
36	21	98	Affected	+51
37	21	87	Affected with whitish nebulosity	+80
38	21	87	Affected with whitish nebulosity	+83
39	14	69	Much affected	+81
40	21	69	Very much affected ; and many faint nebulæ suspected	+71
41	56	76	Affected with very faint nebulosity	+56

2. The fields indicated in the two tables have the following characteristic densities: I=faintest, V=thickest.

Herschel No. 32.—The general density is IV–V. The densest portions V are the centre and the *np* quadrant, the thinnest IV is the *f* limit of the field.

The nebula N.G.C. 3034, which forms a convenient guide of this field, is not mentioned by Roberts, although its figure and surrounding stars show distinctly on the *sf* margin of his plate. The nebula is recorded in the *Preparatory Catalogue* (Specola Vat. X.), p. 54, where the adjoining field (observed in 1919) is described exactly as above, though the observations were made at an interval of seven years and independently of one another.

No. 33.—The density of the field varies irregularly between IV–V and V, the densest parts lying in the *sp* quadrant and around the guiding star.

No. 34.—The centre of the field has density IV, which increases to IV–V or V in the *nf* quadrant. The rest of the field has only III–IV.

No. 35.—The general density of the nebulosities is IV–V, reaching V only in the *nf* quadrant and becoming thinner =III–IV south of the centre.

No. 36.—This field is unusually uniform in density IV–V, which becomes IV in a few regions *n* and *s* of the centre.

No. 37.—The density varies little between IV–V and V. The nebula N.G.C. 4150 is *nf* the guiding star, and was again observed as a small cluster just as ten years before (*Prep. Cat.*, p. 142).

No. 38.—Field most uniform in density IV–V, except a thinner oval gap III, one-half degree in diameter, extending in a *nf* direction from the 5·8 mag. star *BD*+29° 2275.

There are six nebulæ visible on the print: N.G.C. 4245, 4274, 4278, 4283, 4286, 4314, all in the *sf* quadrant except the first, which precedes them. They are listed in the *Prep. Cat.*, p. 142, where the density of the surrounding field is given as IV–V, in full agreement with the above. The two observations have an interval of six years, and are independent, because the nebulæ could in no case be identified and compared with the *Preparatory Catalogue* during the night.

No. 39.—This field is exceptionally dense, =V, with hardly any regions =IV–V. Notice that Herschel calls it "much affected," and that its Gal. Lat. is +81°.

Herschel No. 40.—The general density is IV–V, diminishing to IV from the centre towards the guiding star and increasing to V on the *f* margin of the field. Of Herschel's "suspected" nebulæ a few seem to be indicated on the print, but only one could be ascertained. Its position is approximately $14^h\ 5^m.5+34°\ 6'$ (1860). It seems to be attached to a 15 mag. star; brightness $<12^N$.

No. 41.—The density of this field is throughout only IV–V, or even IV.

Vatican Observatory:
1926 *June.*

Occultation of Venus by the Moon, observed on 1924 *September* 24 *at the Engelhardt Observatory.* By A. Jakovkin.

Greenwich Mean Time. h m s	
13 54 33·0	Contact of southern cusp of Venus and of the bright limb of the Moon.
13 54 45·0	Contact of northern cusp of Venus and of the bright limb of the Moon.
13 55 5·7	The inner contact (Venus disappearing).

The sky was cloudy at reappearance.
Telescope used: 306 mm. Grubb refractor.
Power: 130.

Engelhardt Observatory,
Kasan, U.S.S.R.:
1926 *June* 1.